
Mechanical Design of Overhead Lines

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Mechanical Design of Overhead Lines

Electric power can be carried either by underground cables or overhead transmission and distribution lines. The underground cables are not typically used for power transmission due to two reasons.

1. Power is carried over long distances to remote load centres. Obviously, the installation costs for underground transmission will be huge.
2. Electric power has to be transferred at high voltages for economic reasons. It is very difficult to achieve proper insulation to the cables to withstand higher pressures.

Therefore, power transfer over long distances is done by using overhead lines. With the power demand increase and consequent voltage level rise, power transmission by overhead lines has assumed significant importance.

Nevertheless, an overhead line is subjected to various weather conditions and other external interferences. This asks for the use of adequate mechanical safety factors in order to ensure the continuity of line operation. Typically, the strength of the line needs to be such so it can withstand the worst probable weather conditions. This course focuses on the different aspects of mechanical design of overhead lines.

Overhead Line Main Components

An overhead line may be used to transfer or distribute electric power. The proper overhead line operation depends to a big extent upon its mechanical design. While constructing an overhead line, it has to be verified that line mechanical strength is such so as to provide against the most probable weather conditions. Typically, the main elements of an overhead line are:

- Conductors which transfer power from the sending end station to the receiving end station.
- Supports which may be poles or towers. They keep the conductors at an appropriate level above the earth.

- Insulators that are connected to supports and insulate the conductors from the earth.
- Cross arms which give support to the insulators.
- Miscellaneous elements such as phase plates, danger plates, surge arrestors, etc.

The overhead line operation continuity depends upon the judicious selection of above elements. Hence, it is beneficial to have detailed discussion on them.

Overhead Line Conductor Materials

The conductor is one of the crucial items as most of the financial outlay is invested for it. Hence, correct selection of conductor material and size is of significant importance. The conductor material used for transmission and distribution of electric power needs to have the following characteristics:

- High tensile strength in order to sustain mechanical stresses
- High electrical conductivity
- Low specific gravity so that weight per unit volume is small
- Low cost so that it can be used for considerable distances

All above demands cannot be found in a single material. Hence, while choosing a conductor material for a particular application, a compromise is made between the cost and the needed electrical and mechanical characteristics.

Typically used conductor materials

Typically used conductor materials for overhead lines are copper, aluminium, steel-cored aluminium, galvanised steel and cadmium copper. The selection of a particular material is dependant on the cost, the needed electrical and mechanical characteristics and the local conditions. All conductors used for overhead lines are

typically stranded in order to increase the flexibility. In stranded conductors, there is typically one central wire and around it, successive layers of wires containing 6, 12, 18, 24 wires. Therefore, if there are n layers, the overall number of individual wires is $3n(n+1)+1$. In the production process of stranded conductors, the consecutive layers of wires are twisted or spiralled in different directions so that layers are bound together.

- Copper. Copper is perfect material for overhead lines owing to its great electrical conductivity and increased tensile strength. It is typically used in the hard drawn form as stranded conductor. Even though hard drawing slightly decreases the electrical conductivity, it considerably increases the tensile strength. Copper has great current density. For example, the current carrying capacity of copper per unit of cross-sectional area is significant. This leads to two benefits. Firstly, smaller conductor cross-sectional area is needed and secondly, the area offered by the conductor to wind loads is decreased. Also, this metal is homogeneous, durable and has big scrap value. There is no doubt that copper is perfect material for electric power transmission and distribution. Nevertheless, due to its big cost and non-availability, it is not often used for these purposes. Current trend is to use aluminium instead of copper.
- Aluminium. Aluminium is cheap and light in comparison to copper but it has considerably smaller conductivity and tensile strength. The relative comparison of the two materials is as follows:
 - o The aluminium conductivity is 60% that of copper. The lower aluminium conductivity means that for any specific transmission efficiency, the conductor cross-sectional area must be bigger in aluminium than in copper. For the same resistance, the aluminium conductor diameter is around 1.26 times the copper conductor diameter. The increased aluminium cross-section exposes a bigger surface to wind pressure and, hence, supporting towers have to be designed for greater transverse strength. Typically, this requires the use of higher towers with consequence of bigger sag.
 - o The aluminium specific gravity (2.71 gm/cc) is lower than that of copper (8.9 gm/cc). Hence, an aluminium conductor has almost one-half the weight of

equivalent copper conductor. Due to this, the supporting structures for aluminium need not be made so strong as that of copper conductor.

- Aluminium conductor being light is liable to bigger swings and therefore bigger cross-arms are needed.
- Due to lower tensile strength and bigger co-efficient of linear expansion of aluminium, the sag is bigger in aluminium conductors.

Considering the overall characteristics that include cost, conductivity, tensile strength, weight etc., aluminium has an edge over copper. Hence, it is being predominantly used as a conductor material. It is especially profitable to use aluminium for heavy-current transmission where the conductor size is big and its cost forms a significant proportion of the total cost of complete installation.

- Steel-cored aluminium. Due to low tensile strength, aluminium conductors have bigger sag. This forbids their application for bigger spans and makes them unsuitable for long distance transmission. In order to improve the tensile strength, the aluminium conductor is strengthened with a core of galvanised steel wires. The obtained composite conductor is known as steel-cored aluminium or ACSR (aluminium conductor steel reinforced). Steel-cored aluminium conductor has galvanised steel central core surrounded by a number of aluminium strands. Typically, diameter of both steel and aluminium wires is the same. Typically, the cross-section of the two metals are in the ratio of 1:6 but can be modified to 1:4 in order to get more conductor tensile strength. Figure 1. presents steel-cored aluminium conductor having one steel wire surrounded by six aluminium wires. The result of this composite conductor is that steel core takes bigger percentage of mechanical strength while aluminium strands transfer the bulk of current.

The steel-cored aluminium conductors have the following benefits:

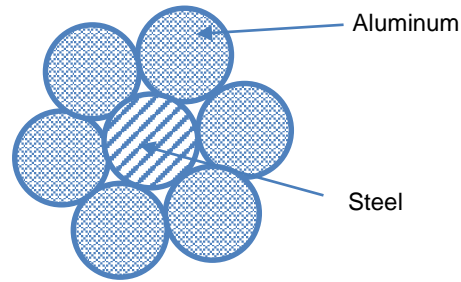


Figure 1. Steel-cored aluminium conductor having one steel wire surrounded by six aluminium wires

- The reinforcement with steel improves the tensile strength but at the same time keeps the composite conductor light. Hence, steel-cored aluminium conductors will create smaller sag and therefore longer spans can be used.
- Due to smaller sag with steel-cored aluminium conductors, towers of smaller heights can be installed.
- Galvanised steel. Steel has considerable tensile strength. Hence, galvanised steel conductors can be applied for long spans or for short line sections exposed to significantly high stresses due to climatic conditions. They are considered as very suitable in rural locations where cheapness is the main issue. Due to steel poor conductivity and high resistance, such conductors are not appropriate for transferring large power over a long distance. Nevertheless, they can be used to advantage for transferring a small power over a small distance where the size of the copper conductor desirable from economic considerations would be too small and therefore inappropriate for use because of poor mechanical strength.
- Cadmium copper. The conductor material now being used in specific installations is copper alloyed with cadmium. An addition of 1% or 2% cadmium to copper improves the tensile strength by roughly 50% and the conductivity is only decreased by 15% below that of pure copper. Hence, cadmium copper conductor can be useful for extremely long spans. Nevertheless, due to cadmium high cost, such conductors will be economical only for lines of small cross-section i.e., where the cost of conductor material is relatively small in

comparison with the support cost.

Line Supports

The supporting structures for overhead line conductors are different pole and tower types called line supports. Typically, the line supports should have the following characteristics:

- Light in weight without the loss of mechanical strength
- Big mechanical strength to sustain the conductor weight and wind loads etc.
- Longer life span
- Easy conductor accessibility for maintenance
- Cheap in cost and economical to service

The line supports used for electric power transmission and distribution are of different types including wooden poles, steel poles, RCC poles and lattice steel towers. The selection of supporting structure for a specific case is dependent upon the line span, cross-sectional area, line voltage, cost and local circumstances.

- Wooden poles. They are made of seasoned wood and are appropriate for lines of moderate cross-sectional area and of shorter spans, say up to 50 metres. Such supports are cheap, easily available, provide insulating features and, hence, are widely used for distribution applications in rural locations as an economical proposition. Typically, the wooden poles tend to rot below the earth level, causing foundation failure. In order to avoid this, the portion of the pole below the earth level is impregnated with preservative substances like creosote oil. Double pole arrangements of the 'A' or 'H' type are typically used (Figure 2.) to obtain a bigger transverse strength than could be economically provided by means of single poles. The main disadvantages to wooden supports are :
 - o Tendency to rot below the earth level

- Relatively smaller life (20-25 years)
- Cannot be used for voltages above 20 kV
- Decreased mechanical strength
- Need occasional inspection

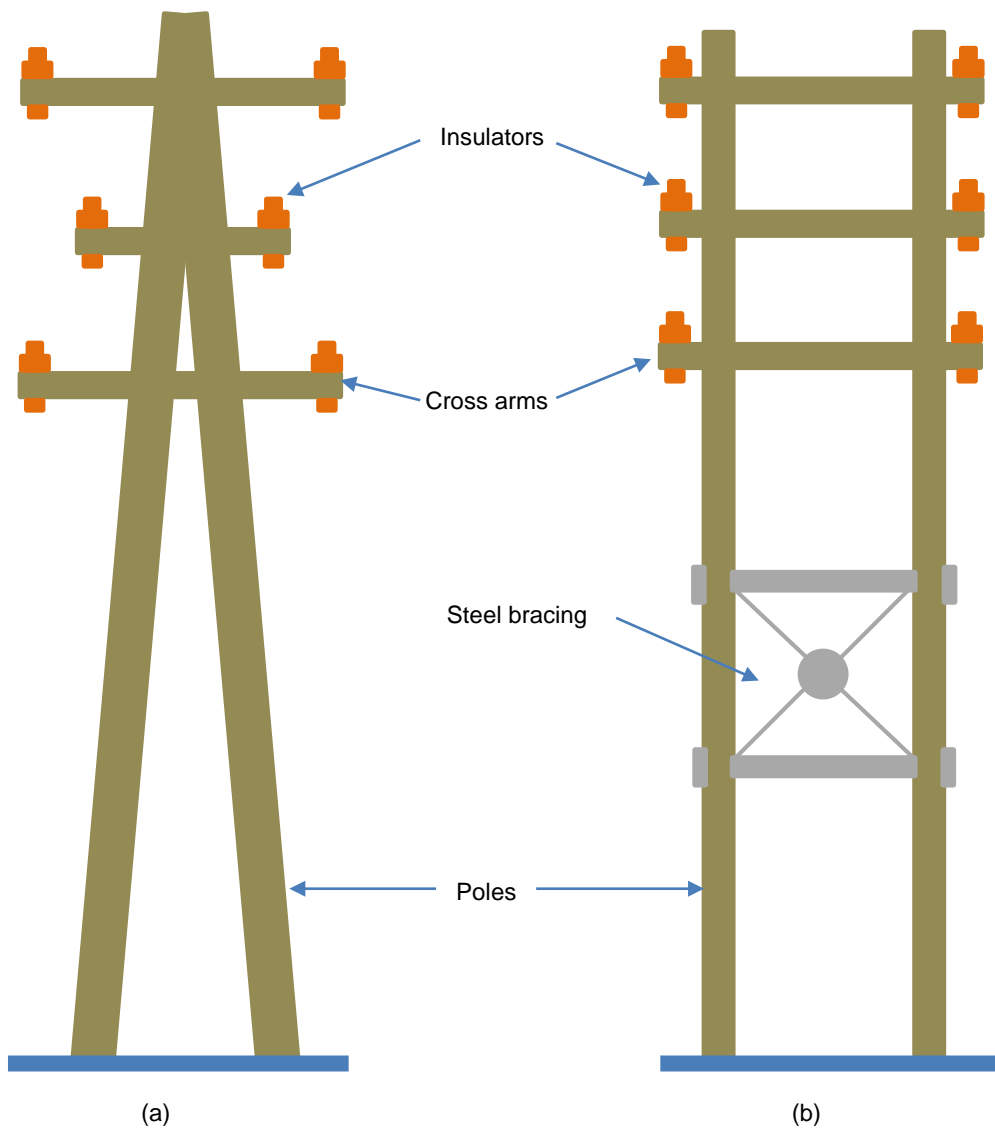
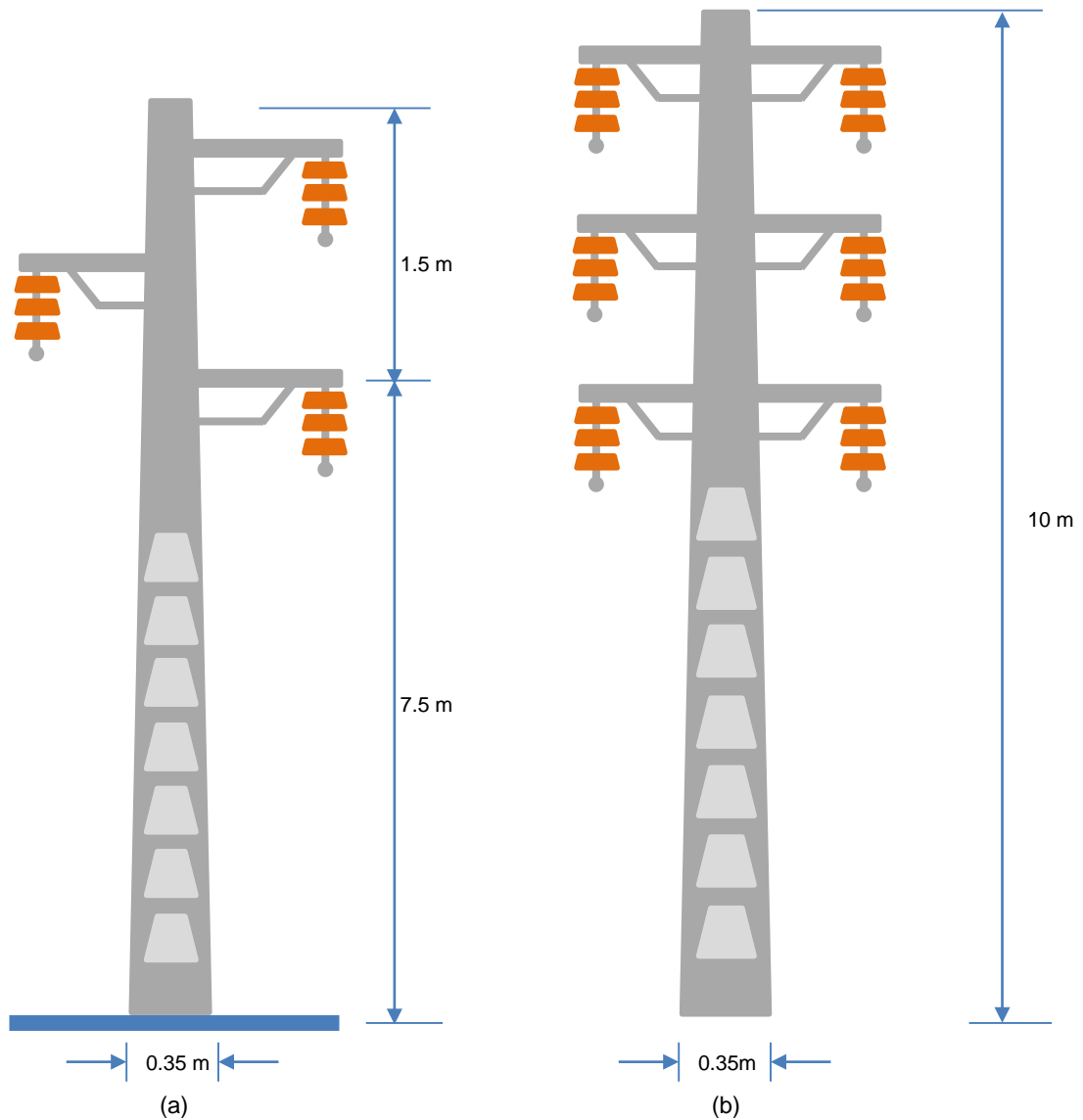


Figure 2. Wooden poles

- Steel poles. The steel poles are typically used as a substitute for wooden poles. They have bigger mechanical strength, longer life and allow longer spans. Typically, such poles are used for distribution purposes in the cities. These supports need to be galvanised or painted in order to extend their life. The steel

poles are of three types:

- Rail poles
 - Tubular poles
 - Rolled steel joints
-
- RCC poles. The reinforced concrete poles have recently become popular as line supports. They have bigger mechanical strength, longer life and allow longer spans than steel poles. Nevertheless, they give good outlook, need little maintenance and have good insulating features. Figure 3 presents RCC poles for single and double circuit. The holes in the poles allow climbing of poles and at the same time decrease the line support weight. The main issue with the use of these poles is the high transport cost owing to their heavy weight. Hence, such poles are typically produced at the site in order to avoid big transportation cost.
 - Steel towers. In reality, wooden, steel and reinforced concrete poles are used for distribution installations at low voltages, say up to 11 kV. Nevertheless, for long distance transmission at higher voltage, steel towers are invariably used. Steel towers have bigger mechanical strength, longer life, can sustain most severe climatic conditions and allow the use of longer spans. The risk of interrupted operation due to broken or punctured insulation is significantly decreased owing to longer spans. Typically, tower footings are earthed by driving rods into the ground. This decreases the lightning troubles as each tower acts as a lightning conductor. Figure 4(a) shows a single circuit tower. Nevertheless, at a moderate extra cost, double circuit tower can be provided as presented in Figure 4(b). The double circuit has the benefit that it ensures continuity of supply. In situation there is breakdown of one circuit, the continuity of supply can be kept by the other circuit.



Insulators

The overhead line conductors need to be supported on the poles or towers in such a way that conductor currents do not flow to ground through supports for example, line conductors have to be adequately insulated from supports. This is accomplished by securing line conductors to supports with the help of insulators. The insulators give necessary insulation between line conductors and supports and therefore prevent any leakage current from conductors to ground. Typically, the insulators need to have the following desirable features:

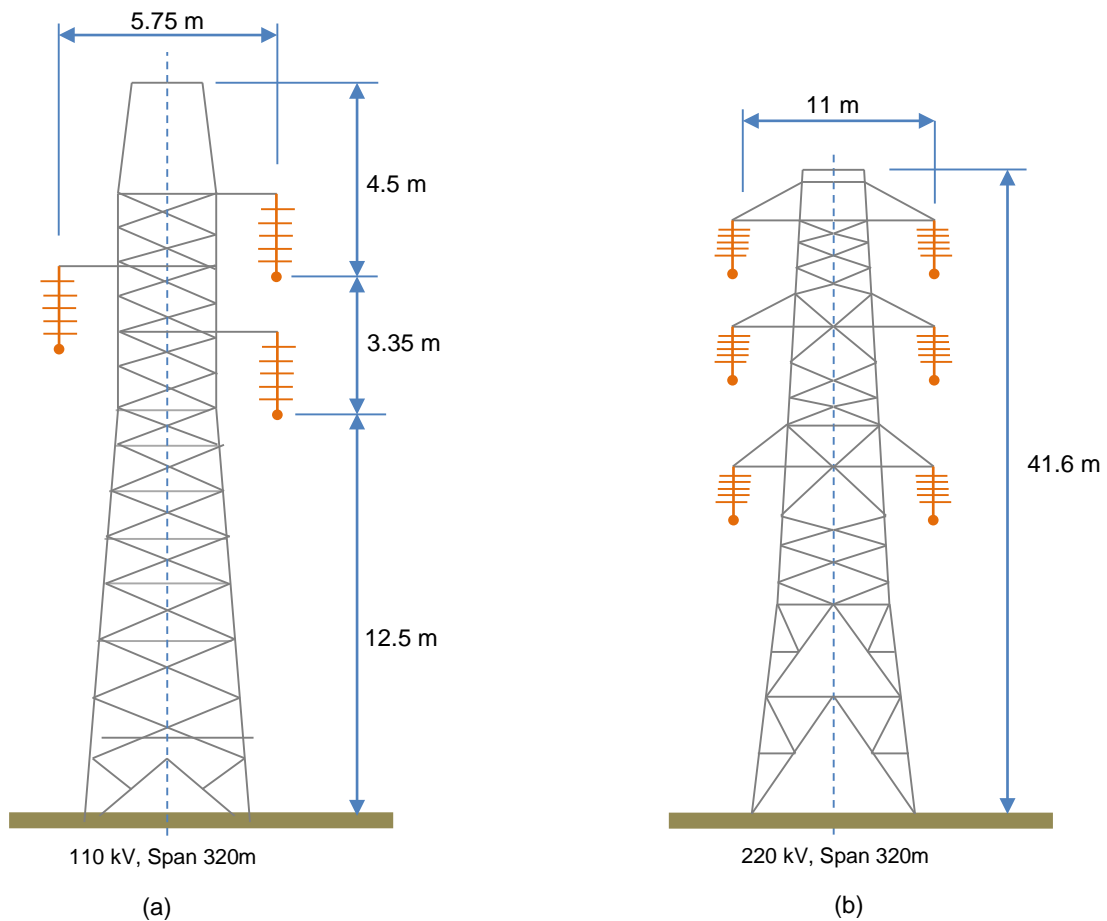


Figure 4. Steel towers (a) single circuit (b) double circuit

- Big mechanical strength in order to sustain conductor load, wind load etc.
- Big insulator material electrical resistance in order to avoid leakage currents to ground.
- Big insulator material relative permittivity in order that dielectric strength is high.
- The insulator material needs to be non-porous, free from impurities and cracks otherwise the permittivity will be decreased.
- Big ratio of puncture strength to flashover.

The most typically used material for insulators of overhead line is porcelain but glass, steatite and specific composition materials are also applied to a certain extent. Porcelain is made by firing at a high temperature a mixture of kaolin, feldspar and quartz. It is mechanically stronger than glass, gives less trouble from leakage and is

less impacted by temperature changes.

Insulator Types

The proper overhead line operation depends to a significant extent upon the adequate selection of insulators. There are few insulator types but the most typically used are pin type, suspension type, strain insulator and shackle insulator.

- Pin type insulators. The section of a pin type insulator is presented in Figure 5. As the name implies, the pin type insulator is linked to the pole cross-arm. There is a groove on the insulator upper end for housing the conductor. The conductor goes through this groove and is bound by the annealed wire of the same material as the conductor. Pin type insulators are used for electric power transmission and distribution at voltages up to 33 kV. Above operating voltage of 33 kV, the pin type insulators become too bulky and therefore uneconomical. Insulators are required to sustain both mechanical and electrical stresses. The electrical stress is caused by line voltage and may cause the insulator breakdown. The insulator electrical breakdown can happen either by flash-over or puncture. In flashover, an arc happens between the line conductor and insulator pin (i.e., ground) and the discharge jumps across the air gaps, following shortest distance. Figure 6 presents the arcing distance ($a+b+c$) for the insulator. In case of flash-over, the insulator will continue to act in its adequate capacity unless extreme heat generated by the arc destroys the insulator. In case of puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown happens, the insulator is completely destroyed due to significant heat. In reality, proper thickness of porcelain is provided in the insulator to prevent puncture by the line voltage. The ratio of puncture strength to flashover voltage is known as safety factor:

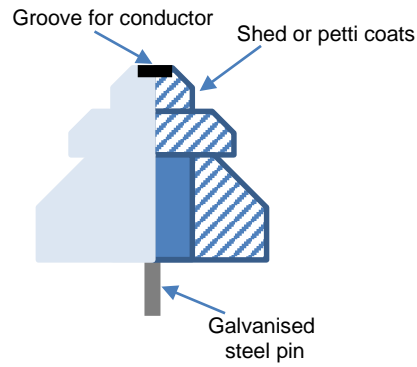


Figure 5. Pin-type insulator

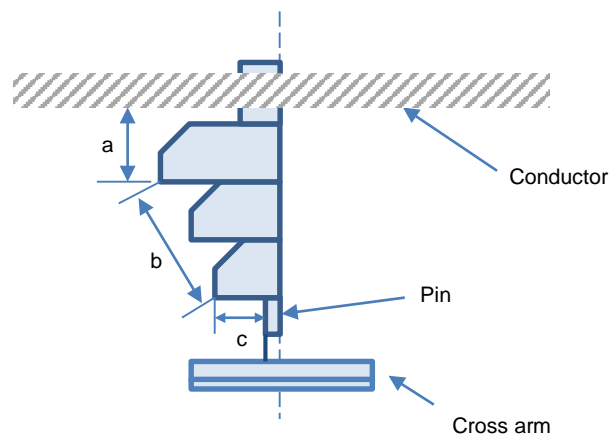


Figure 6. Arcing distance

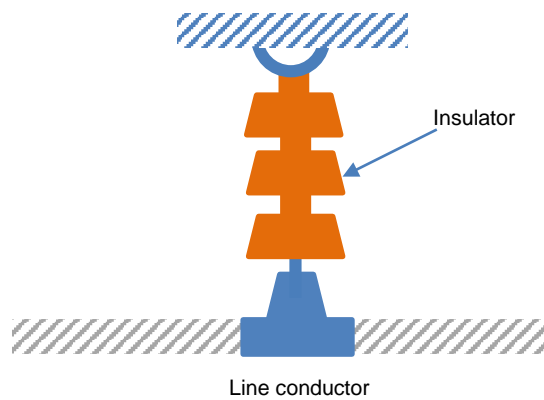


Figure 7. Suspension type insulator

$$\text{Safety factor of insulator} = \frac{\text{Puncture strenght}}{\text{Flash - over voltage}}$$

It is preferable that the value of safety factor is big so that flash-over takes place before the insulator gets punctured. For pin type insulators, the value of safety factor is around

10.2.

- Suspension type insulators. The cost of pin type insulator quickly increases as the working voltage is increased. Hence, this insulator type is not economical beyond 33 kV. For high voltages (>33 kV), it is a typical practice to use suspension type insulators presented in Figure 7. They consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the tower cross-arm. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series depends upon the working voltage. For example, if the working voltage is 66 kV, then six discs in series will be needed in the string.

Suspension insulator advantages are:

- Suspension type insulators are cheaper than pin type insulators for voltages above 33 kV.
- Each unit or disc of suspension type insulator is made for low voltage, typically 11 kV.
- Depending upon the working voltage, the required number of discs can be connected in series.
- If any disc is destroyed, the whole string does not become useless because the damaged disc can be replaced by the new one.
- The suspension arrangement gives line bigger flexibility. The connection at the cross arm is such that insulator string is free to swing in any direction and can take up the position where mechanical stresses are minimum.
- In case of increased power demand on the transmission line, it is found better to provide the bigger power demand by increasing the line voltage than to provide extra set of conductors. The extra insulation needed for

the increased voltage can be easily obtained in the suspension arrangement by adding the needed number of discs.

- The suspension type insulators are typically used with steel towers. Since the conductors run below the tower grounded cross-arm, this arrangement gives partial protection from lightning.
- Strain insulators. When there is a dead end of the line or there is corner or sharp curve, the line is subjected to bigger tension. In order to relieve the line of excessive tension, strain insulators are installed. For low voltage lines (< 11 kV), shackle insulators are used as strain insulators. Nevertheless, for high voltage lines, strain insulator consists of an assembly of suspension insulators as presented in Figure 8. The discs of strain insulators are installed in the vertical plane. When the tension in lines is exceedingly big, as at long river spans, two or more strings are installed in parallel.

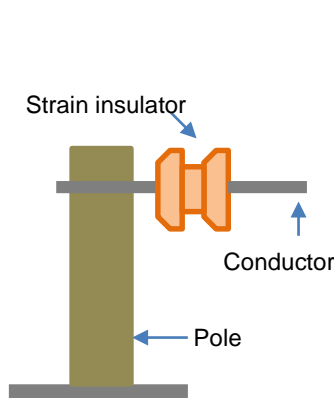


Figure 8. Strain insulators

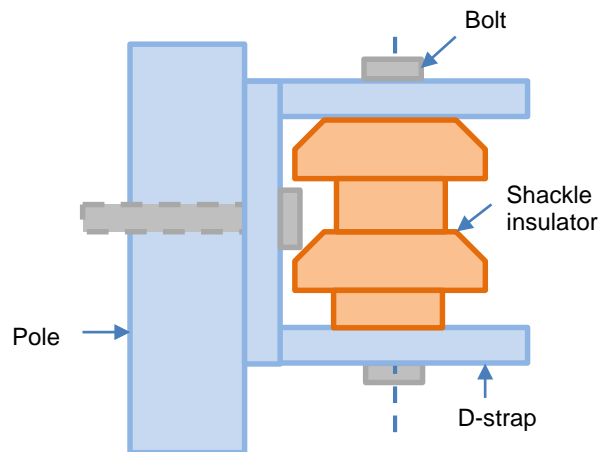


Figure 9. Shackle insulators

- Shackle insulators. The shackle insulators were used as strain insulators in the past. These days, they are normally used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly connected to the pole with a bolt or to the cross arm. Figure 9 presents a shackle insulator fixed to the pole. The conductor in the groove is linked with a soft binding wire.

Potential Distribution over Suspension Insulator String

A string of suspension insulators consists of porcelain discs connected in series through metallic links. Figure 10(a) presents 3-disc string of suspension insulators. The porcelain part of each disc is in between two metal links. Hence, each disc forms a capacitor C as presented in Figure 10(b). This is known as mutual capacitance or self-capacitance. If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each element would have been the same for example, $V/3$ as presented in Figure 10(b). Nevertheless, in reality, capacitance also exists between metal fitting of each disc and tower or ground. This is known as shunt capacitance C_1 . Due to shunt capacitance, charging current is not the same through all the discs of the string as shown in Figure 10(c). Hence, voltage across each disc will be different. Apparently, the disc closest to the line conductor will have the maximum voltage. Therefore, referring to Figure 10(c), V_3 will be higher than V_2 or V_1 .

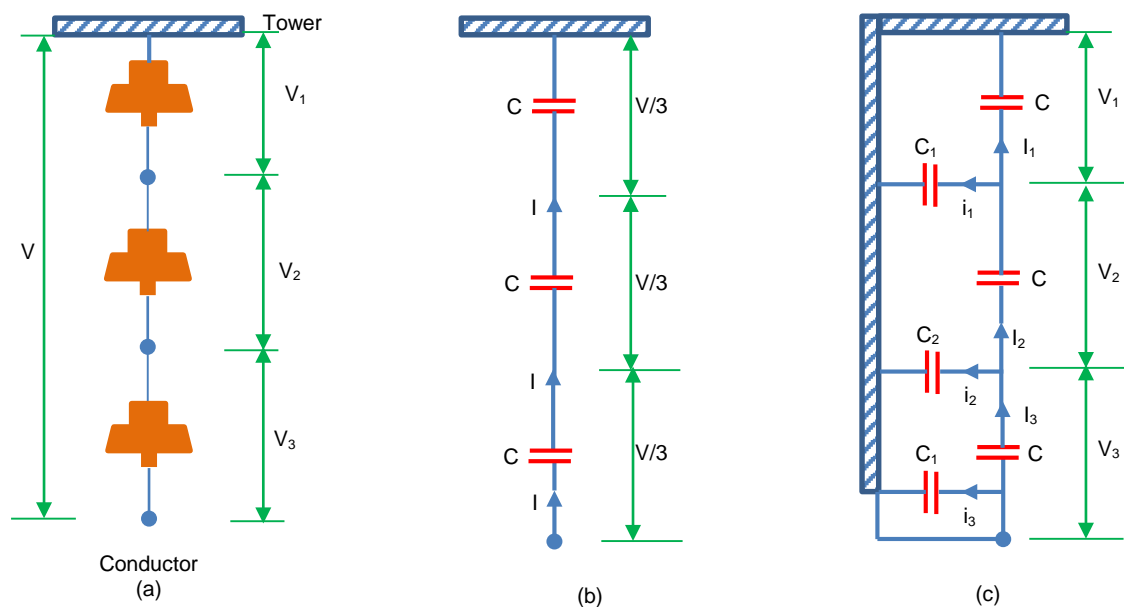


Figure 10. Disc equivalent circuit

The following points have to be noted regarding the potential distribution over a string of suspension insulators:

- The disc closest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc continues to decrease.

- The voltage impressed on a string of suspension insulators does not equally distribute itself across the individual discs due to the shunt capacitance
- If the voltage impressed across the string was DC, then voltage across each unit would be the equal, since insulator capacitances are ineffective for DC.
- The unit closest to the conductor is under maximum electrical stress and is likely to be punctured. Hence, methods have to be provided to equalise the potential across each element.

String Efficiency

As mentioned above, the voltage applied across the string of suspension insulators is not equally distributed across different elements or discs. The disc closest to the conductor has much bigger potential than the other discs. This one-sided potential distribution is not desirable and is typically expressed in terms of string efficiency. The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc closest to the conductor is known as string efficiency:

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

Where: n - number of discs in the string

String efficiency is vital consideration since it defines the potential distribution along the string. The higher the string efficiency, the more uniform is the voltage distribution. Therefore, 100% string efficiency is an ideal case for which the voltage across each disc will be exactly the same. Even though it is not possible to reach 100% string efficiency, efforts need to be made to enhance it as close to this value as possible. Figure 11 presents the equivalent circuit for a 3-disc string. Let us suppose that self-capacitance of each disc is C. Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance for example, $C_1 = KC$. Starting from the cross-arm or tower, the voltage across each element is V_1, V_2 and V_3 respectively. Using Kirchhoff's current law at node A, we get,

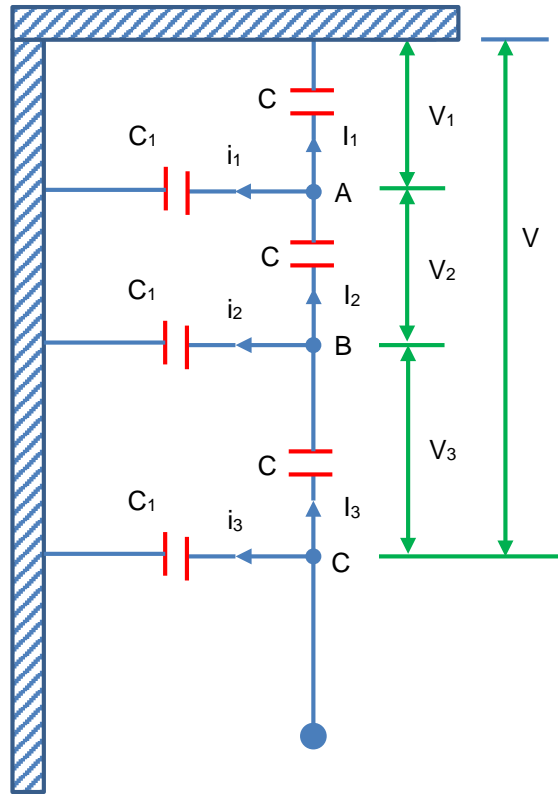


Figure 11. Equivalent circuit for a 3-disc string

$$I_2 = I_1 + i_1$$

Or
$$V_2 \omega C = V_1 \omega C + V_1 \omega C_1$$

Or
$$V_2 \omega C = V_1 \omega C + V_1 \omega KC$$

$$V_2 = V_1(1 + K) \quad (1)$$

Using Kirchhoff's current law at node B, we get,

$$I_3 = I_2 + i_2$$

Or
$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega C_1$$

Or
$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) \omega KC$$

Or
$$V_3 = V_2 + (V_1 + V_2)K$$

$$= KV_1 + V_2(1 + K)$$

$$= KV_1 + V_1(1 + K)^2$$

$$= V_1[K + (1 + K)^2]$$

$$V_3 = V_1[1 + 3K + K^2] \quad (2)$$

Voltage between conductor and ground (for example, tower) is

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= V_1 + V_1(1 + K) + V_1(1 + 3K + K^2) \\ &= V_1(3 + 4K + K^2) \end{aligned}$$

$$V = V_1(1 + K)(3 + K) \quad (3)$$

From formulas (1), (2), (3), we get,

$$\frac{V_1}{1} = \frac{V_2}{1+K} = \frac{V_3}{1+3K+K^2} = \frac{V}{(1+K)(3+K)} \quad (4)$$

Voltage across top element, $V_1 = \frac{V}{(1+K)(3+K)}$

Voltage across second element from top, $V_2 = V_1(1 + K)$

Voltage across third element from top, $V_3 = V_1(1 + 3K + K^2)$

$$\begin{aligned} \%age \ string \ efficiency &= \frac{voltage \ across \ string}{n \times voltage \ across \ disc \ nearest \ to \ conductor} \times 100 \\ &= \frac{V}{3 \times V_3} \times 100 \end{aligned}$$

The following points have to be noted from the previous mathematical assessment:

- If for example, $K=0.2$, then from formula (4), we get, $V_2=1.2V_1$ and $V_3=1.64 V_1$. This indicates that disc closest to the conductor has maximum voltage across it. The voltage across other discs decreasing progressively as the cross-arm in reaches.
- The higher the value of $K (=C_1/C)$, the more unequal is the potential across the discs and lower is the string efficiency.
- The difference in voltage distribution rises with the bigger number of discs. Hence, shorter string has bigger efficiency than the larger one.

Ways of Enhancing String Efficiency

It has been previously seen that potential distribution in a string of suspension insulators is not equal. The maximum voltage appears across the insulator closest to the line conductor and progressively decreases as the cross arm is reached. If the insulation of the most stressed insulator (closest to conductor) breaks down or flash over takes occurs, the breakdown of other elements will happen in succession. Therefore it is important to equalise the potential across the different string elements in order to improve the string efficiency. The different methods for this are:

- By using longer cross-arms. The string efficiency is dependant upon the value of K . For example, ratio of shunt capacitance to mutual capacitance. The lower the value of K , the higher is the string efficiency and more uniform is the voltage distribution. The value of K can be decreased by decreasing the shunt capacitance. In order to decrease shunt capacitance, the distance of conductor from tower needs to be increased for example, longer cross-arms need to be used. Nevertheless, limitations of tower cost and strength do not allow the use of very long cross-arms. In reality, $K=0.1$ is the limit that can be reached by this method.
- By grading the insulators. In this approach, insulators of various dimensions are selected in a way that each has a different capacitance. The insulators are capacitance graded for example, they are assembled in the string in such a way that the top element has the minimum capacitance, growing progressively as the bottom element (closest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalise the potential distribution across the string elements. This approach has the disadvantage that a big number of different-sized insulators are needed. Nevertheless, good results can be found by using standard insulators for most of the string and bigger elements for that close to the line conductor.
- By using a guard ring. The potential across each element in a string can be equalised by applying a guard ring which is a metal ring electrically linked to the conductor and surrounding the bottom insulator as presented in the Figure 13.

The guard ring introduces capacitance between metal parts and the line conductor. The guard ring is contoured in a way that shunt capacitance currents i_1, i_2 etc. are equal to metal fitting line capacitance currents i_1', i_2' etc. The result is that same charging current I flows through each string element. Finally, there will be equal potential distribution across the elements.

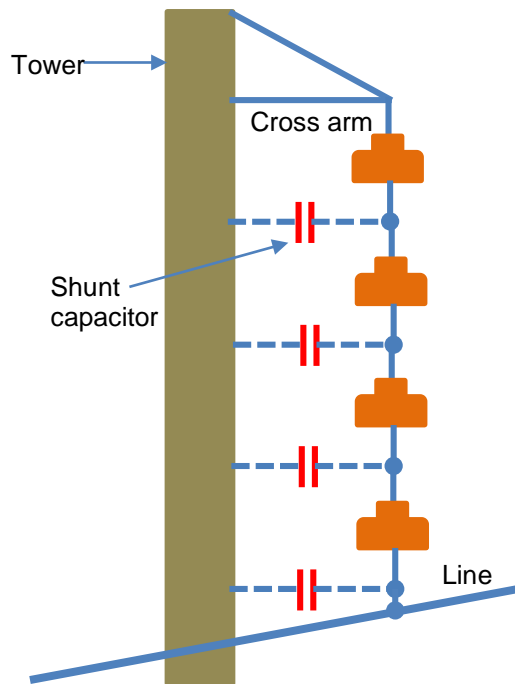


Figure 12. Insulator shunt capacitance

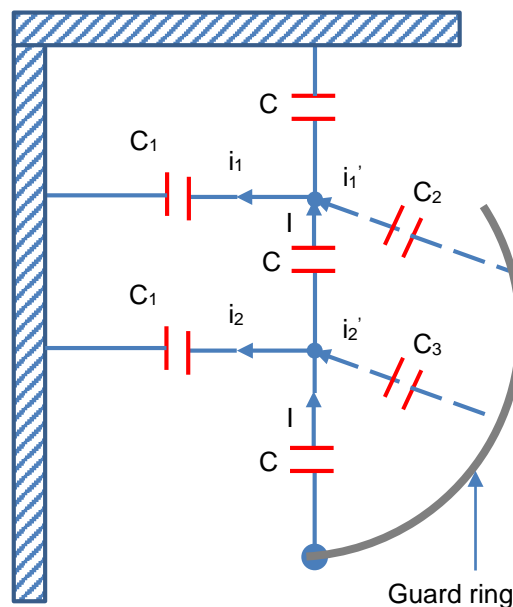


Figure 13. Guard ring

Important Issues

While solving issues relating to string efficiency, the following points have to be considered and taken into consideration:

- The maximum voltage appears across the disc closest to the conductor (phase conductor).
- The voltage across the string is equal to phase voltage. For example, voltage across string=Voltage between line and ground=Phase Voltage

$$\text{Line voltage} = \sqrt{3} \times \text{Voltage across string}$$

Example 1. In a 33 kV overhead distribution line, there are three elements in the string of insulators. If the capacitance between each insulator pin and ground is 11% of self-capacitance of each insulator, calculate (a) voltage distribution over 3 insulators and (b) string efficiency.

- Solution. Figure 14 presents the equivalent circuit of string insulators. Let V_1 , V_2 and V_3 be the voltage across top, middle and bottom unit. If C is the self-capacitance of each element, then KC will be the shunt capacitance.

$$K = \frac{\text{shunt capacitance}}{\text{Self - capacitance}} = 0.11$$

Voltage across string, $V = \frac{33}{\sqrt{3}} = 19.05 \text{ kV}$

At point A

$$I_2 = I_1 + i_1$$

Or $V_2 \omega C = V_1 \omega C + V_1 K \omega C$

Or $V_2 = V_1(1 + K) = V_1(1 + 0.11)$

Or $V_2 = 1.11 V_1$

At point B

$$I_3 = I_2 + i_2$$

Or $V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$

Or $V_3 = V_2 + (V_1 + V_2) K$
 $= 1.11 V_1 + (V_1 + 1.11 V_1) 0.11$

Or $V_3 = 1.342 V_1$

Voltage across the complete string is:

$$V = V_1 + V_2 + V_3 = V_1 + 1.11V_1 + 1.342V_1 = 3.452 V_1$$

Or $19.05 = 3.452 V_1$

Voltage across top element, $V_1 = \frac{19.05}{3.452} = 5.52 \text{ kV}$

Voltage across middle element $V_2 = 1.11 V_1 = 1.11 \times 5.52 = 6.13 \text{ kV}$

Voltage across bottom element, $V_3 = 1.342 V_1 = 1.342 \times 5.52 = 7.4 \text{ kV}$

$$\text{String efficiency} = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{19.05}{3 \times 7.4} \times 100 = 85.8\%$$

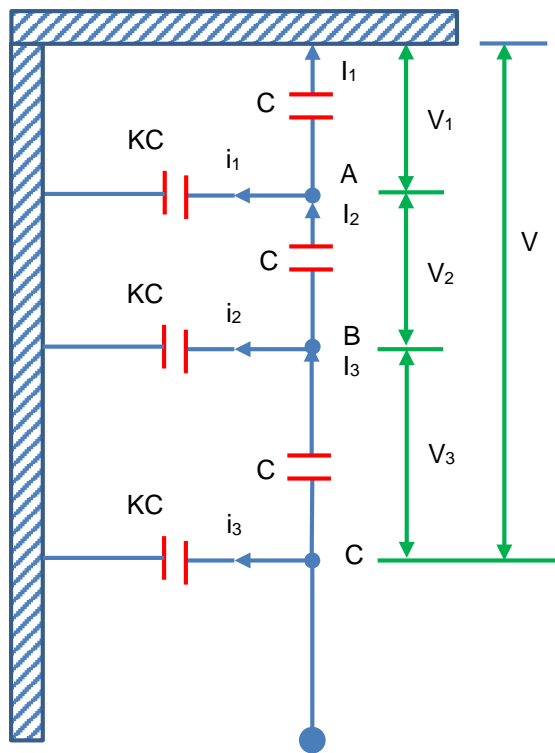


Figure 14. Equivalent circuit for three string insulators

Example 2. A 3-phase transmission overhead line is supported by three disc insulators. The potentials across top element (close to the tower) and middle unit are 8 kV and 11 kV respectively. Find (a) the ratio of capacitance between pin and ground to the self-capacitance of each element (b) The line voltage and (c) string efficiency.

- Solution. The equivalent circuit of string insulators is presented in Figure 14. It is given that $V_1=8$ kV and $V_2=11$ kV.

(i) Let K be the ratio of capacitance between pin and ground to self-capacitance. If C is the self-capacitance of each element, then capacitance between pin and ground = KC . Using Kirchoff's current law to point A,

At point A:

$$I_2 = I_1 + i_1$$

Or
$$V_2\omega C = V_1\omega C + V_1K\omega C$$

Or
$$V_2 = V_1(1 + K)$$

Or
$$K = \frac{V_2 - V_1}{V_1} = \frac{11 - 8}{8} = 0.375$$

At point B:

$$I_3 = I_2 + i_2$$

Or
$$V_3\omega C = V_2\omega C + (V_1 + V_2)K\omega C$$

Or
$$V_3 = V_2 + (V_1 + V_2)K = 11 + (8 + 11) \times 0.375 = 18.12 \text{ kV}$$

Voltage between line and ground = $V_1 + V_2 + V_3 = 8 + 11 + 18.12 = 37.12 \text{ kV}$

Line voltage = $\sqrt{3} \times 37.12 = 64.28 \text{ kV}$

$$\text{String efficiency} = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{37.12}{3 \times 18.12} \times 100 = 68.28\%$$

Example 3. Each line of a 3-phase system is supported by a string of 3 similar insulators. If the voltage across the line unit is 17.5 kV, find the line to neutral voltage.

Assume that the shunt capacitance between each insulator and ground is 1/8th of the capacitance of the insulator itself. Also calculate the string efficiency.

- Solution. Figure 15 presents the equivalent circuit of string insulators. If C is the self-capacitance of each element, then KC will be the shunt capacitance where $K=1/8=0.125$. Voltage across line element, $V_3=17.5$ kV

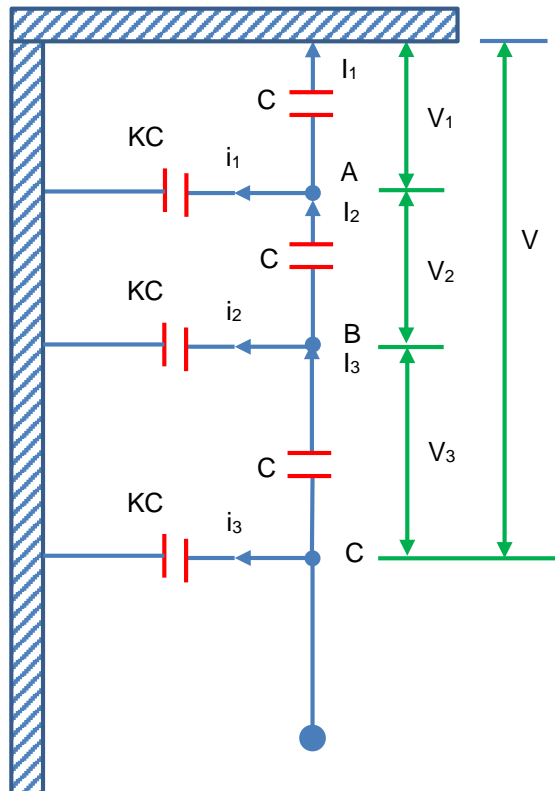


Figure 15. Equivalent circuit of string insulators

At point A:

$$I_2 = I_1 + i_1$$

Or

$$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

Or

$$V_2 = V_1(1 + K) = V_1(1 + 0.125)$$

Or

$$V_2 = 1.125 V_1$$

At point B:

$$I_3 = I_2 + i_2$$

Or
$$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

Or
$$V_3 = V_2 + (V_1 + V_2) K$$

$$= 1.125 V_1 + (V_1 + 1.125 V_1) \times 0.125$$

Or
$$V_3 = 1.39 V_1$$

Voltage across top element,
$$V_1 = \frac{V_3}{1.39} = \frac{17.5}{1.39} = 12.59 \text{ kV}$$

Voltage across middle element
$$V_2 = 1.125 V_1 = 1.125 \times 12.59 = 14.16 \text{ kV}$$

Voltage between line and ground (i.e., line to neutral)

$$V_1 + V_2 + V_3 = 12.59 + 14.16 + 17.5 = 44.25 \text{ kV}$$

$$\text{String efficiency} = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{19.05}{3 \times 7.4} \times 100 = 85.8\%$$

$$\text{String efficiency} = \frac{44.25}{3 \times 17.5} \times 100 = 84.28\%$$

Example 4. The three bus-bar conductors in an outdoor substation are supported by elements of post type insulators. Each element consists of a stack of 3 pin type insulators linked one on the top of the other. The voltage across the lowest insulator is 13.1 kV and that across the next element is 11 kV. Find the bus-bar voltage of the station.

- Solution. The equivalent circuit of insulators is the presented in Figure 15. It is shown that $V_3=13.1\text{kV}$ and $V_2=11 \text{ kV}$. Let K be the ratio of shunt capacitance to self-capacitance of each element.

Using Kirchhoff's current law to points A and B, we can derive the following formulas:

$$V_2 = V_1(1 + K)$$

Or

$$V_1 = \frac{V_2}{1+K} \quad (1)$$

and

$$V_3 = V_2 + (V_1 + V_2)K \quad (2)$$

Replacing the value of $V_1 = V_2/(1 + K)$ in formula (2), we get,

$$V_3 = V_2 + \left[\frac{V_2}{1 + K} + V_2 \right] K$$

Or

$$\begin{aligned} V_3(1 + K) &= V_2(1 + K) + [V_2 + V_2(1 + K)]K \\ &= V_2[(1 + K) + K + (K + K^2)] \\ &= V_2(1 + 3K + K^2) \end{aligned}$$

$$13.1(1 + K) = 11[1 + 3K + K^2]$$

Or

$$11K^2 + 19.9K - 2.1 = 0$$

Solving this formula, we get, $K=0.1$

$$V_1 = \frac{V_2}{1 + K} = \frac{11}{1 + 0.1} = 10 \text{ kV}$$

Voltage between line and ground = $V_1 + V_2 + V_3 = 10 + 11 + 13.1 = 34.1 \text{ kV}$

Voltage between bus-bars (i.e., line voltage)

$$= 34.1 \times \sqrt{3} = 59 \text{ kV}$$

Example 5. An insulator string has three elements, each having a safe working voltage of 15 kV. The ratio of self-capacitance to shunt capacitance of each element is 8:1. Calculate the string's maximum safe working voltage. Also calculate the string efficiency.

- Solution. The equivalent circuit of string insulators is presented in Figure 15. The maximum voltage appears across the lowest unit in the string.

$$V_3=15 \text{ kV}; K=1/8=0.125$$

Using Kirchhoff's current law at point A, we get,

$$V_2=V_1(1+K)$$

$$\text{or } V_1 = V_2 / (1 + K) = V_2 / (1 + 0.125) = 0.89V_2 \quad \dots(1)$$

Using Kirchhoff's current law at point B, we get,

$$V_3 = V_2 + (V_1 + V_2)K = V_2 + (0.89V_2 + V_2) \times 0.125$$

$$V_3 = 1.236 V_2$$

$$\text{Voltage across middle element, } V_2 = \frac{V_3}{1.236} = \frac{15}{1.236} = 12.13 \text{ kV}$$

$$\text{Voltage across top element, } V_1 = 0.89V_2 = 0.89 \times 12.13 = 10.79 \text{ kV}$$

$$\text{Voltage across the string, } = V_1 + V_2 + V_3 = 10.79 + 12.13 + 15 = 37.92 \text{ kV}$$

$$\text{String efficiency, } = \frac{37.92}{3 \times 15} \times 100 = 84.26\%$$

Example 6. A string of 4 insulators has a self-capacitance equal to 10 times the pin to ground capacitance. Calculate (a) the voltage distribution across different elements expressed as a percentage of overall voltage across the string and (b) string efficiency.

- Solution. When the number of insulators in a string surpasses 3, the nodal equation method becomes tedious. In those cases, there is a simple way to solve the problem. In this method, shunt capacitance (C_1) and self-capacitance (C) of each insulator are modelled by their equivalent reactances. As it is only the ratio of capacitances which defines the voltage distribution, the problem can be simplified by assigning unity value to X_C for example, assuming $X_C = 1\Omega$.

If ratio of $C/C_1 = 10$, then we have $X_C = 1\Omega$ and $X_{C_1} = 10\Omega$.

(a) Assume $X_C = 1\Omega$. As the ratio of self-capacitance to shunt capacitance (for example, C/C_1) is 10, hence, $X_{C_1} = 10\Omega$ as presented in Figure 16 (a). Assume that potential V across the string is such that 1A current goes in the top insulator. Now the potential across each insulator can be calculated. Therefore:

$$\text{Voltage across top element, } V_1 = 1\Omega \times 1 \text{ A} = 1 \text{ volt}$$

$$\text{Voltage across second element, } V_2 = 1\Omega \times 1.1 \text{ A} = 1.1 \text{ volts}$$

Voltage across third element, $V_3=1\Omega \times 1.31 \text{ A} = 1.31 \text{ volts}$

Voltage across fourth element, $V_4=1\Omega \times 1.65 \text{ A} = 1.65 \text{ volts}$

Voltage obtained across the string, $V = 1 + 1.1 + 1.31 + 1.65 = 5.06 \text{ volts}$

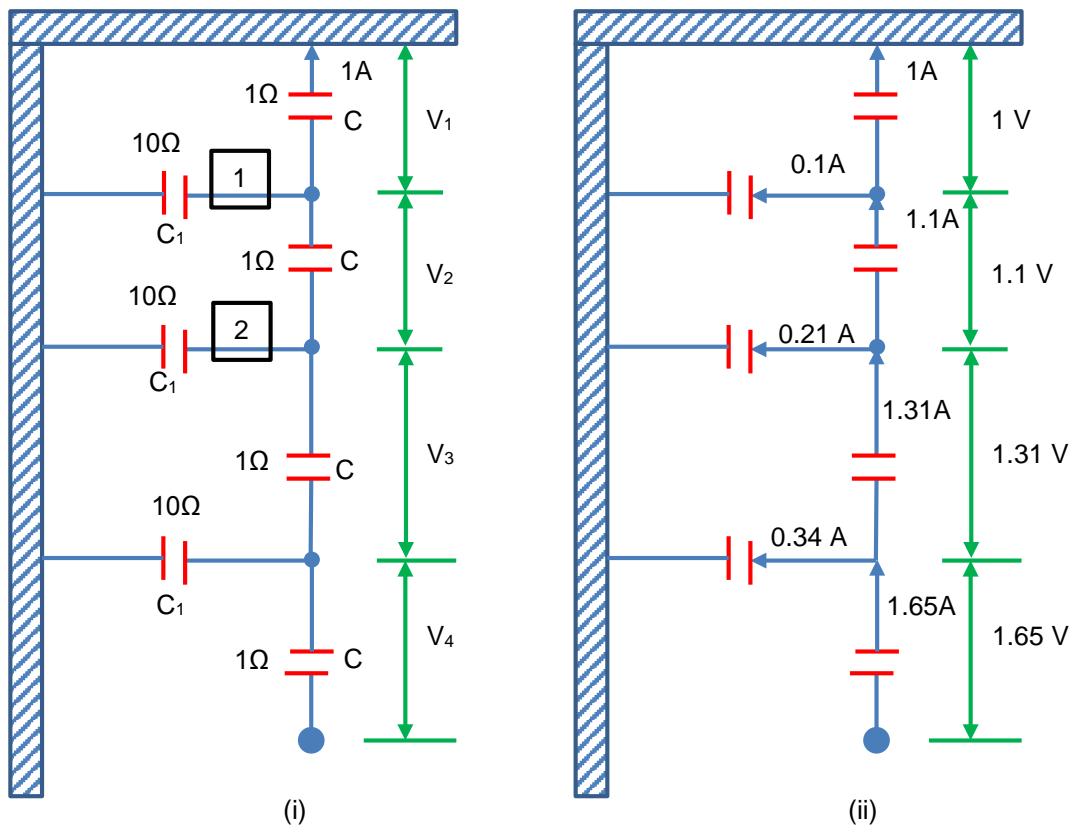


Figure 16. Equivalent circuit of string insulators

The voltage across each element expressed as a percentage of V (for example 5.06 volts) becomes:

Top element $= \left(\frac{1}{5.06}\right) \times 100 = 19.76\%$

Second from top $= \left(\frac{1.1}{5.06}\right) \times 100 = 21.74\%$

Third from top $= \left(\frac{1.31}{5.06}\right) \times 100 = 25.9\%$

Fourth from top $= \left(\frac{1.65}{5.06}\right) \times 100 = 32.6\%$

$$\text{String efficiency} = \frac{V}{4 \times V_4} \times 100 = \frac{5.06}{4 \times 1.65} \times 100 = 76.6\%$$

Example 7. A string of 5 insulators is connected across a 100 kV line. If the capacitance of each disc to ground is 0.1 of the insulator capacitance, find (a) the voltage distribution on the insulator discs and (b) the string efficiency.

- Solution. Suppose $X_C=1\Omega$. As the ratio of self-capacitance to shunt capacitance is 10, $X_{C1}=10\Omega$ as presented in Figure 17(a). Assume that potential V across the string is such that 1A current runs in the top insulator. Then potential across each insulator will be as presented in Figure 17(b).

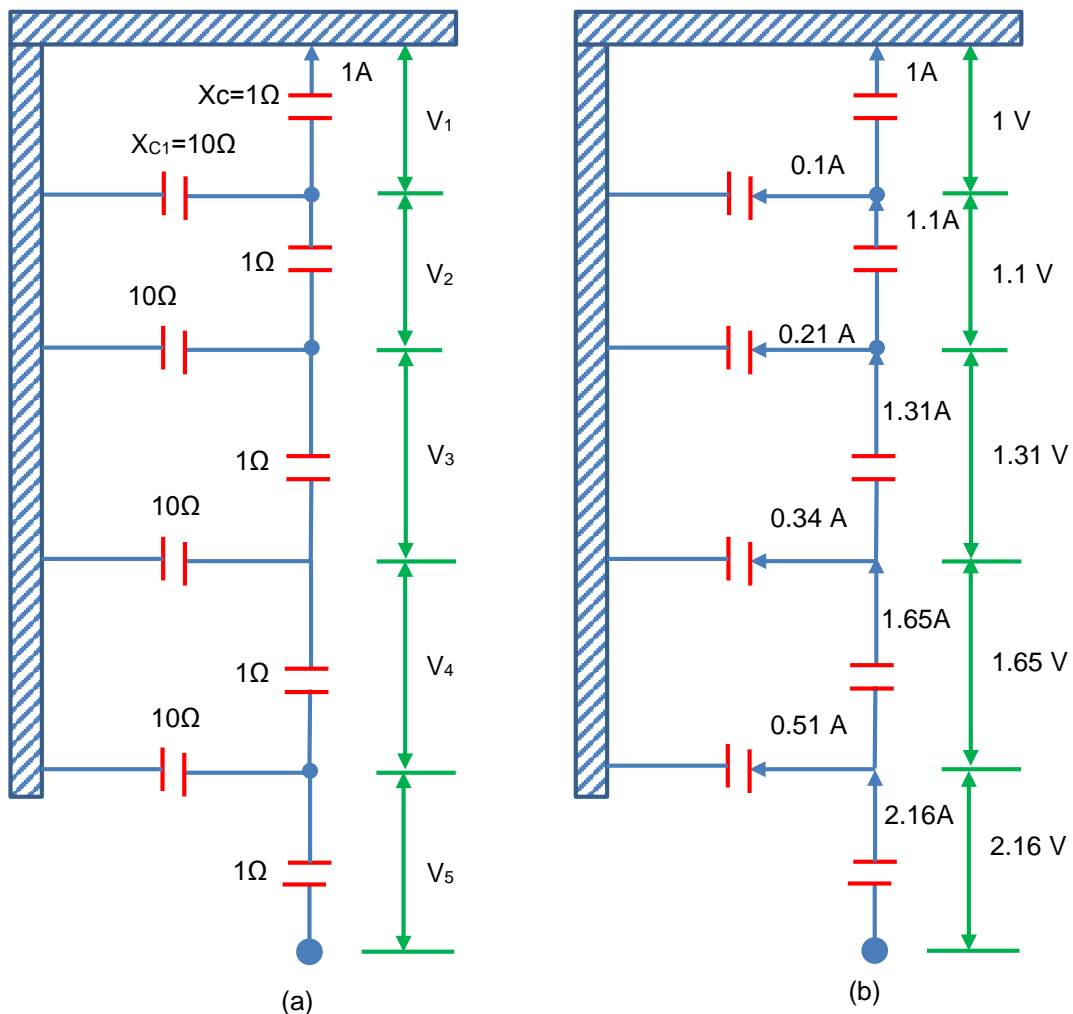


Figure 17. Equivalent circuit of string insulators

The value obtained for $V=1+1.1+1.31+1.65 + 2.16=7.22$ volts and starting from top, the percentage of V (for example, 7.22 volts) across different elements are:

- 13.8 %, 15.2 %, 18.2 %, 22.8 % and 30%

$$\text{Voltage across string} = \frac{100}{\sqrt{3}} = 57.7 \text{ kV}$$

$$\text{Voltage across top insulator, } V_1 = 0.138 \times 57.7 = 7.96 \text{ kV}$$

$$\text{Voltage across 2}^{\text{nd}} \text{ from top, } V_2 = 0.152 \times 57.7 = 8.77 \text{ kV}$$

$$\text{Voltage across 3}^{\text{rd}} \text{ from top, } V_3 = 0.182 \times 57.7 = 10.5 \text{ kV}$$

$$\text{Voltage across 4}^{\text{th}} \text{ from top, } V_4 = 0.228 \times 57.7 = 13.16 \text{ kV}$$

$$\text{Voltage across 5}^{\text{th}} \text{ from top, } V_5 = 0.3 \times 57.7 = 17.3 \text{ kV}$$

$$\text{String efficiency} = \frac{57.7}{5 \times 17.3} \times 100 = 66.7\%$$

Example 8. Each conductor of a 3-phase high-voltage transmission line is supported by a string of 4 suspension type disc insulators. If the potential difference across the second element from top is 13.2 kV and across the third from top is 18 kV, find the voltage between conductors.

- Solution. Assume $X_C=1\Omega$. If K is the ratio of shunt-capacitance to self-capacitance, then $X_{C1}=1/K \Omega$ as presented in Figure 18 (a). Assume voltage across string is such that current in top insulator disc is 1 A. Then voltage across each insulator can be found as seen in Figure 18(b).

Therefore, the voltage across first shunt capacitance from top is 1V and its reactance is $1/K \Omega$. Hence, current through it is K ampere. Therefore, current through second insulator from top is $(1+K)$ A and voltage across it is $(1+K) \times 1 = (1+K)$ V.

With reference to Figure 18 (b), it can be concluded that:

$$\frac{V_2}{V_1} = \frac{1+k}{1}$$

Or

$$V_2 = V_1(1 + K) \quad (1)$$

Also

$$\frac{V_3}{V_1} = \frac{(1 + 3K + K^2)}{1}$$

$$V_3 = V_1(1 + 3K + K^2) \quad (2)$$

Dividing (2) by (1) it can be concluded:

$$\frac{V_3}{V_2} = \frac{1 + 3K + K^2}{1 + K}$$

It is given that $V_3 = 18 \text{ kV}$ and $V_2 = 13.2 \text{ kV}$

$$\frac{18}{13.2} = \frac{1 + 3K + K^2}{1 + K}$$

Or $13.2K^2 + 21.6K - 4.8 = 0$

Solving this formula, we get, $K=0.2$.

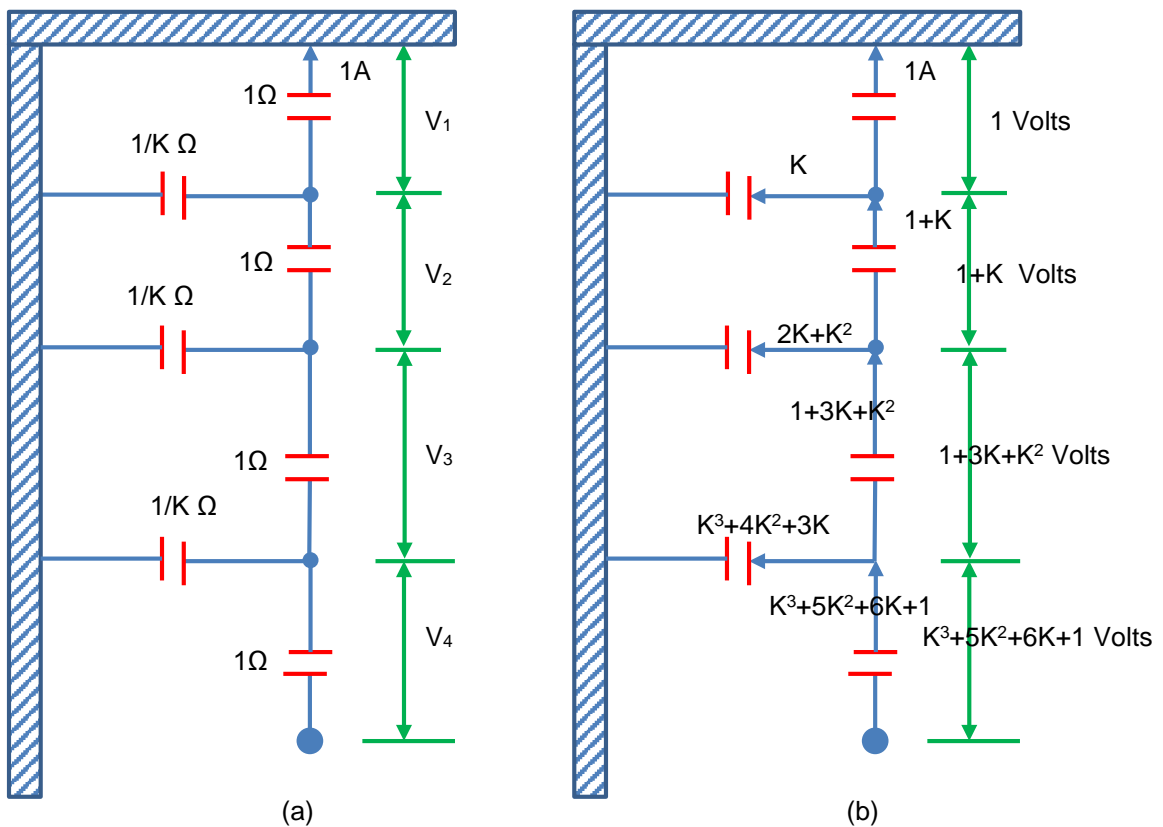


Figure 18. Equivalent circuit of string insulators

$$V_1 = \frac{V_2}{(1 + K)} = \frac{13.2}{1.2} = 11 \text{ kV}$$

$$V_4 = V_1(1 + K^3 + 5K^2 + 6K) = 11(1 + 0.008 + 0.2 + 1.2) = 26.49 \text{ kV}$$

Voltage between line and ground (for example, line voltage)

$$= V_1 + V_2 + V_3 + V_4$$

$$= 11 + 13.2 + 18 + 26.49 = 68.69 \text{ kV}$$

Voltage between conductors (for example, line voltage)

$$68.69 \times \sqrt{3} = 119 \text{ kV}$$

Example 9. A string of four insulators has a self-capacitance equal to 5 times pin to ground capacitance. Calculate (a) the voltage distribution across different elements as a percentage of overall voltage across the string and (b) string efficiency.

- Solution. The ratio of self-capacitance (C) to pin-ground capacitance (C₁) is C/C₁=5. Assume X_C=1Ω. Then X_{C1}=5Ω. Assume the voltage V across string is such that current in the top insulator is 1A as presented in Figure 19 (a). The potential across different insulators will be as presented in Figure 19 (b).

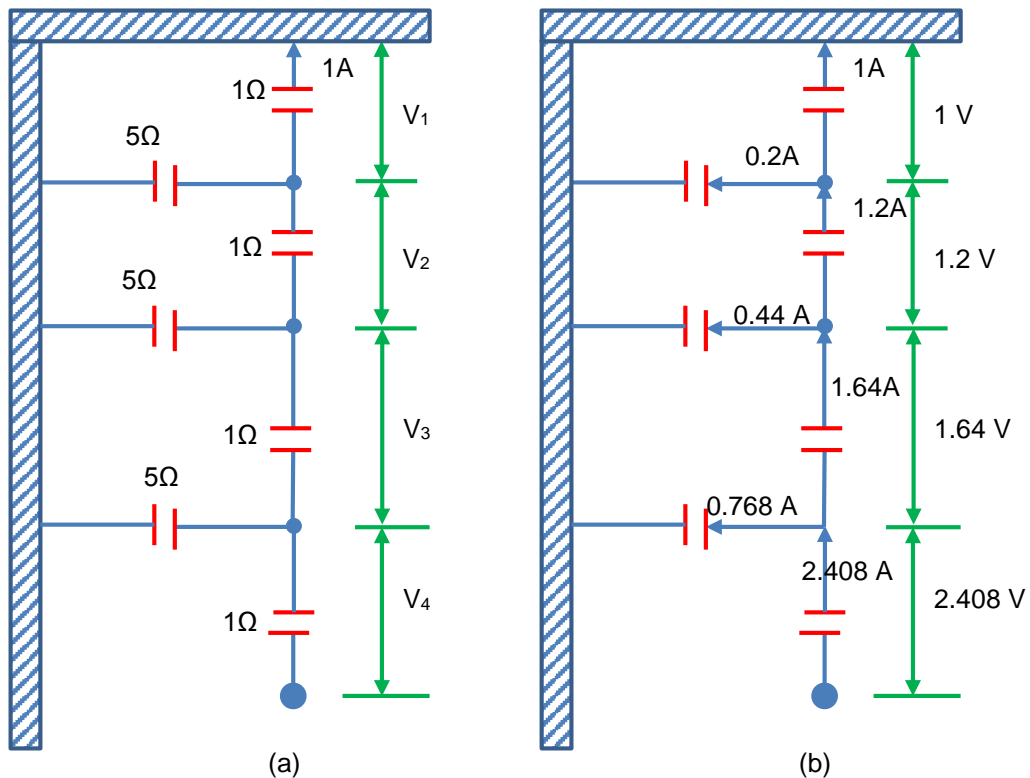


Figure 19. Equivalent circuit of string insulators

The voltage found across the string is expressed as:

$$V = 1 + 1.2 + 1.64 + 2.408 = 6.248 \text{ volts}$$

The voltage across each element expressed as a percentage of V (for example, 6.248 V) is expressed by:

$$\text{Top element} = \frac{1}{6.248} \times 100 = 16\%$$

$$\text{Second from top,} = \frac{1.2}{6.248} \times 100 = 19.2\%$$

$$\text{Third from top,} = \frac{1.64}{6.248} \times 100 = 26.3\%$$

$$\text{Fourth from top,} = \frac{2.408}{6.248} \times 100 = 38.5\%$$

$$\text{String efficiency} = \frac{6.248}{4 \times 2.408} \times 100 = 64.86\%$$

Example 10. The self-capacitance of each element in a string of three suspension insulators is C. The shunting capacitance of the connecting metal work of each insulator to ground is 0.15 C while for line it is 0.1 C. Find (a) the voltage across each insulator as a percentage of the line voltage to ground and (b) string efficiency.

- Solution. In an actual string of insulators, three capacitances exist, self-capacitance of each insulator, shunt capacitance and capacitance of each element to line as presented in Figure 20 (a). Nevertheless, capacitance of each element to line is very small and is typically neglected. Figure 20 (b) presents the equivalent circuit of string insulators.

- At point A:

$$I_2 + i'_1 = I_1 + i_1$$

-

$$\text{Or} \quad V_2 \omega C + (V_2 + V_3) 0.1 \omega C = V_1 \omega C + 0.15 C V_1 \omega$$

-

$$\text{Or} \quad 0.1 V_3 = 1.15 V_1 - 1.1 V_2$$

-

$$\text{Or} \quad V_3 = 11.5 V_1 - 11 V_2 \quad (1)$$

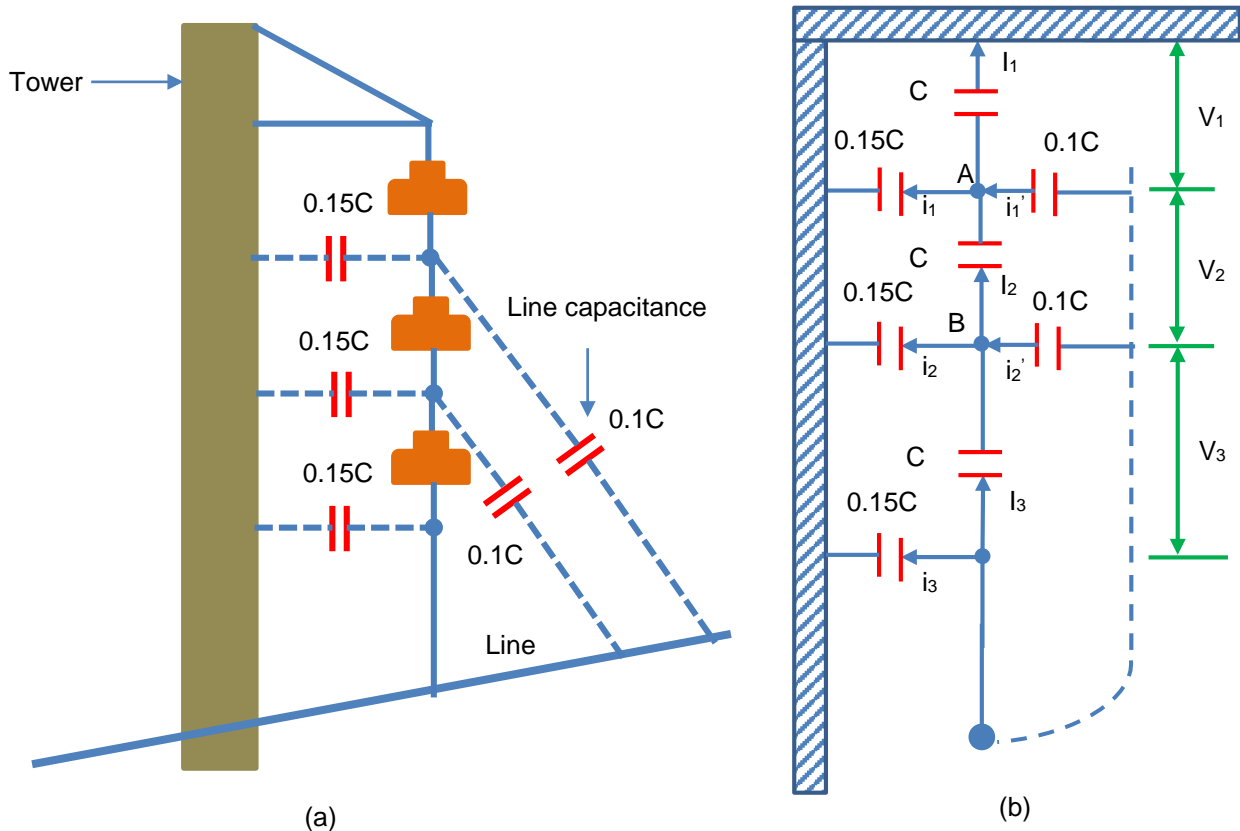


Figure 20. Equivalent circuit of string insulators

At point B:

$$I_3 + i_2' = I_2 + i_2$$

Or
$$V_3\omega C + V_3 \times 0.1C \times \omega = V_2\omega C + (V_1 + V_2)\omega \times 0.15C$$

Or
$$1.1V_3 = 1.15V_2 + 0.15V_1$$

Replacing the value of V_3 from expression (a) into expression (b), we get,

$$= 1.1(11.5 V_1 - 11V_2) = 1.15V_2 + 0.15V_1 \quad (2)$$

Or

$$13.25V_2 = 12.5V_1$$

Or

$$V_2 = \frac{12.5}{13.25} V_1 \quad (3)$$

Replacing the value of V_2 from expression (3) into expression (1), it can be found:

$$V_3 = 11.5V_1 - 11 \left(\frac{12.5V_1}{13.25} \right) = \left(\frac{14.8}{13.25} \right) V_1$$

Now voltage between conductor and ground is:

$$V = V_1 + V_2 + V_3 = V_1 \left(1 + \frac{12.5}{13.25} + \frac{14.8}{13.25} \right) = \left(\frac{40.55V_1}{13.25} \right) \text{ volts}$$

$$V_1 = \frac{13.25V}{40.55} = 0.326 V$$

$$V_2 = \frac{12.5 \times 0.326V}{13.25} = 0.307 V$$

$$V_3 = \frac{14.8 \times 0.326V}{13.25} = 0.364 V$$

(1) The voltage across each element determined as a percentage of V becomes:

$$\text{Top element,} \quad = V_1 \times \frac{100}{V} = 0.326 \times 100 = 32.6\%$$

$$\text{Second from top,} \quad = V_2 \times \frac{100}{V} = 0.307 \times 100 = 30.7\%$$

$$\text{Third from top,} \quad = V_3 \times \frac{100}{V} = 0.364 \times 100 = 36.4\%$$

$$\text{String efficiency} = \frac{V}{3 \times 0.364V} \times 100 = 91.5\%$$

Example 11. Each phase of a 3-phase system is suspended by a string of 3 same insulators of self-capacitance C. The shunt capacitance of connecting metal work of each insulator is 0.2 C to ground and 0.1 C to line. Find the system string efficiency if a guard ring increases the capacitance to the line of metal work of the lowest insulator to 0.3 C.

- Solution. The capacitance between each element and phase is artificially increased by applying a guard ring as presented in Figure 21. This configuration tends to equalise the potential across different elements and therefore leads to enhanced string efficiency. It is given that with the use of guard ring, insulator link-pin capacitance to the line of the lowest element is increased from 0.1 C to 0.3 C.

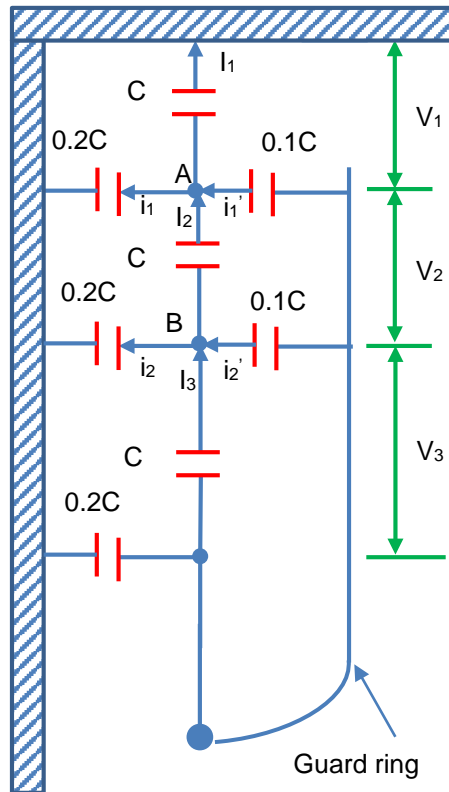


Figure 21. Equivalent circuit of string insulators

At point A:

$$I_2 + i_1' = I_1 + i_1$$

Or

$$\begin{aligned} V_2\omega C + (V_2 + V_3)\omega \times 0.1C \\ = V_1\omega C + V_1 \times 0.2C\omega \end{aligned}$$

$$V_3 = 12V_1 - 11V_2 \quad (1)$$

At point B:

$$I_3 + i_2' = I_2 + i_2$$

Or

$$V_3\omega C + V_3 \times 0.3C \times \omega = V_2\omega C + (V_1 + V_2)\omega \times 0.2C$$

Or

$$1.3V_3 = 1.2V_2 + 0.2V_1$$

Replacing the value of V_3 from expression (1) into expression (2), we find,

$$1.3(12V_1 - 11V_2) = 1.2V_2 + 0.2V_1$$

Or

$$15.5V_2 = 15.4V_1$$

$$V_2 = \frac{15.4V_1}{15.5} = 0.993V_1$$

Replacing the value of V_2 from expression (3) into expression (1), we get,

$$V_3 = 12V_1 - 11 \times 0.993V_1 = 1.077V_1$$

Voltage between conductor and ground (for example, line voltage)

$$= V_1 + V_2 + V_3 = V_1 + 0.993V_1 + 1.077V_1 = 3.07V_1$$

$$\text{String efficiency} = \frac{3.07V_1}{3 \times 1.077V_1} \times 100 = 95\%$$

Example 12. It is needed to grade a string having seven suspension insulators. If the pin to ground capacitance are all same to C , find the phase to pin capacitance that would give the same voltage across each string insulator.

- Solution. Let $C_1, C_2 \dots C_6$ respectively be the required line to pin capacitances of the elements as presented in Figure 22. As the voltage across each insulator has to be the same:

$$I_1 = I_2 = I_3 = I_4 = I_5 = I_6 = I_7$$

At point A:

$$I_2 + i'_1 = I_1 + i_1$$

Or

$$i'_1 = i_1 \quad (I_1 = I_2)$$

Or

$$\omega C_1(6V) = \omega CV \quad (\text{Voltage across } C_1 = 6V)$$

$$C_1 = \frac{C}{6} = 0.167C$$

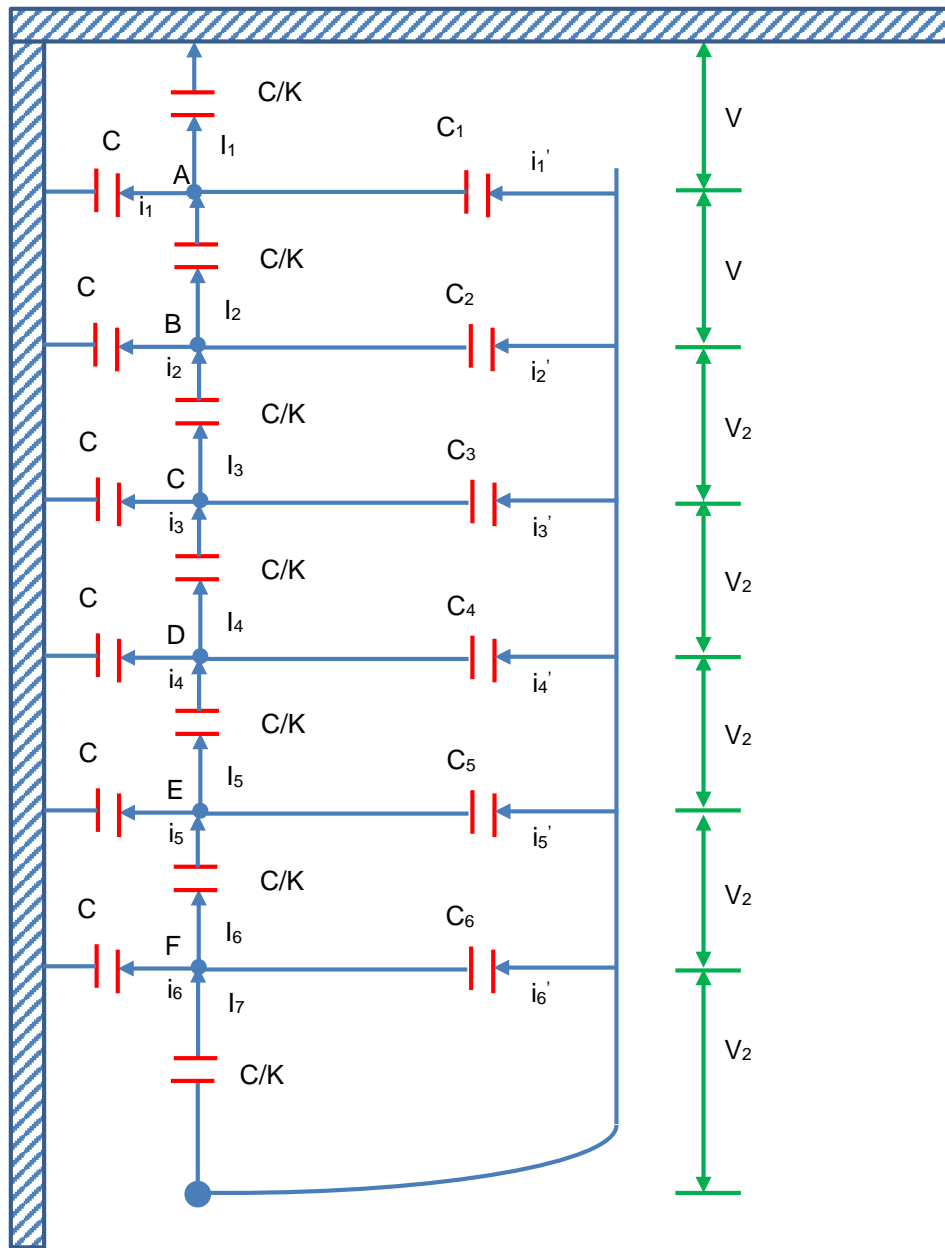


Figure 22. Equivalent circuit of string insulators

At point B:

$$i_2' = i_2$$

Or

$$\omega C_2(5V) = \omega C(2V)$$

$$C_2 = \frac{2C}{5} = 0.4C$$

At point C:

$$i'_3 = i_3$$

Or

$$\omega C_3(4V) = \omega C(3V)$$

$$C_3 = \frac{3C}{4} = 0.75C$$

At point D:

$$i'_4 = i_4$$

Or

$$\omega C_4(3V) = \omega C(4V)$$

$$C_4 = \frac{4C}{3} = 1.33C$$

At point E:

$$i'_5 = i_5$$

Or

$$\omega C_5(2V) = \omega C(5V)$$

$$C_5 = \frac{5C}{2} = 2.5C$$

At point F:

$$i'_6 = i_6$$

Or

$$\omega C_6V = \omega C(6V)$$

$$C_6 = 6C$$

Corona Effect

When an alternating potential difference is brought across two conductors whose spacing is big in comparison to their diameters, there is no apparent change in the condition of atmospheric air surrounding the wires if the introduced voltage is low. Nevertheless, when the applied voltage surpasses a specific value, known as critical

disruptive voltage, the conductors are surrounded by a faint violet glow known as corona.

The corona phenomenon is accompanied by a hissing sound, ozone production, power loss and radio interference. The higher the voltage is raised, the bigger and higher the luminous envelope becomes, and bigger are the sound, the power loss and the radio noise. If the introduced voltage is raised to breakdown value, a flash-over will happen between the conductors due to the insulation breakdown.

The phenomenon of violet glow, hissing noise and ozone gas production in an overhead transmission line is known as corona.

In the case the conductors are polished and smooth, the corona glow will be uniform across the conductor length otherwise the rough points will seem brighter. With DC voltage, there is difference in the appearance of the two wires. The positive wire has uniform glow around it, while the negative conductor has spotty glow.

Corona formation theory

Certain ionisation always exists in air due to cosmic rays, ultraviolet radiations and radioactivity. Hence, in normal situations, the air around the conductors has some ionised elements (for example, free electrons and positive ions) and neutral molecules. When potential difference is applied between the conductors, potential gradient is set up in the air which will have the maximum value at the conductor surfaces. Under the potential gradient impact, the existing free electrons acquire bigger velocities. The bigger the applied voltage, the bigger the potential gradient and more is the velocity of free electrons.

When the conductor potential gradient at surface comes to around 30 kV per cm (maximum value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with sufficient force to dislodge one or more electrons from it. This creates another ion and one or more free electrons, which are accelerated until they collide with other neutral molecules, therefore generating other ions. Therefore, the ionisation process is cumulative. The result of this ionisation is that either corona is made or spark happens between the conductors.

Factors Influencing Corona

The corona phenomenon is impacted by the physical state of the atmosphere as well as by the line conditions. The following are the factors upon which corona depends:

- Atmosphere. As corona is made due to ionisation of air around the conductors, hence, it is impacted by the atmosphere physical conditions. In the stormy weather, the number of ions is more than normal and as such corona happens at lower voltage in comparison to fair weather.
- Conductor size. The corona effect is dependent on the conductor shape and conditions. The rough and irregular surface will give rise to more corona since unevenness of the surface reduces the value of breakdown voltage. Therefore a stranded conductor has irregular surface and therefore gives rise to more corona than a solid conductor.
- Conductor spacing. If the spacing between the conductors is big in comparison to their diameters, there may not be any corona effect. It is because bigger distance between conductors decreases the electro-static stresses at the conductor surface, therefore avoiding corona formation.
- Line voltage. The line voltage significantly impacts corona. If it is low, there is no change in the condition of air surrounding the conductors and therefore no corona is formed. Nevertheless, if the line voltage has such a value that electrostatic stresses created at the conductor surface make the air around the conductor conducting, then corona is formed.

Important Points

The corona phenomenon has an important role in the overhead transmission line design. Hence, it is useful to assess the following terms much used in the corona effect evaluation:

- Critical disruptive voltage. It is the minimum line-neutral voltage at which corona happens.

Imagine two conductors of radii r cm and spaced d cm apart. If V is the line-neutral potential, then potential gradient at the conductor surface is expressed as:

$$q = \frac{V}{r \log_e \frac{d}{r}} \text{ volts/cm}$$

In order that corona is created the value of g must be made equal to the air breakdown strength. The air breakdown strength at 76 cm pressure and temperature of 25°C is 30 kV/cm (max) or 21.2 kV/cm (RMS) and is denoted by q_0 . If V_c is the line-neutral potential needed under these conditions, then:

$$q_0 = \frac{V_c}{r \log_e \frac{d}{r}}$$

q_0 – air breakdown strength at 76 cm of mercury and 25°C=30 kV/cm (max) or 21.2 kV/cm (RMS)

Critical disruptive voltage, $V_c = q_0 r \log_e \frac{d}{r}$

The above formula for disruptive voltage is made under standard conditions for example, at 76 cm of Hg and 25°C. Nevertheless, if these conditions differ, the air density also changes, therefore changing the value of q_0 . The value of q_0 is directly proportional to air density. Therefore, the air breakdown strength at a barometric pressure of b cm of mercury and temperature of t °C becomes δq_0 where

$$\delta = \text{air density factor} = \frac{3 \cdot 92b}{273 + t}$$

Under typical conditions, the value of $\delta=1$.

Critical disruptive voltage, $V_c = q_0 \delta r \log_e \frac{d}{r}$

Correction must also be considered for the conductor surface condition. This is accounted for by multiplying the above formula by irregularity factor m_0 .

Critical disruptive voltage, $V_c = m_0 q_0 \delta r \log_e \frac{d}{r}$ kV/phase

Where

m_0 – 1 for polished conductors

0.98 to 0.92 for dirty conductors

0.87 to 0.8 for stranded conductors

- Visual critical voltage. It is the minimum line-neutral voltage at which corona glow appears all along the line conductors.

It has been noted that in the parallel conductor situations, the corona glow does not start at the disruptive voltage V_c but at a bigger voltage V_v , known as visual critical voltage. The line-neutral effective value of visual critical voltage is expressed by the following empirical equation:

$$V_v = m_v q_0 \delta r \left(1 + \frac{0.3}{\sqrt{\delta r}} \right) \log_e \frac{d}{r} \text{ kV/phase}$$

where m_v is another irregularity factor with a value of 1.0 for polished conductors and 0.72 to 0.82 for rough conductors.

- Power loss due to corona. Formation of corona is typically accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is surpassed, the power loss due to corona is expressed as:

$$P = 242.2 \left(\frac{f + 25}{\delta} \right) \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5} \frac{\text{kW}}{\text{km}} / \text{phase}$$

Where

f - supply frequency in Hz

V – phase-neutral voltage (RMS)

V_c – disruptive voltage (RMS) per phase

Corona Benefits and Disadvantages

Corona has numerous benefits and disadvantages. In the adequate design of a high voltage overhead line, a balance has to be struck between the benefits and disadvantages.

Benefits

- Due to corona creation, the air surrounding the conductor becomes conducting and therefore conductor virtual diameter is increased. The increased diameter decreases the electrostatic stresses between the conductors.
- Corona decreases the effects of transients created by surges.

Disadvantages

- Corona is accompanied by an energy loss. This impacts the line transmission efficiency.
- Ozone is generated by corona and may cause conductor corrosion due to chemical action.
- The current taken by the line due to corona is non-sinusoidal and therefore non-sinusoidal voltage drop happens in the line. This may cause inductive interference with neighbouring communication lines.

Methods of Decreasing Corona Effect

It has been noted that intense corona effects are observed at a working voltage of 33 kV or above. Hence, adequate design has to be made to avoid corona on the substations or bus-bars designed for 33 kV and bigger voltages otherwise highly ionised air may cause flash-over in the insulators or between the lines, causing considerable equipment damage. The corona effects can be decreased by the following actions:

- By increasing conductor size. By increasing conductor size, the voltage at

which corona happens is increased and therefore corona effects are considerably decreased. This is one of the reasons that ACSR conductors which have a bigger cross-sectional area are used in transmission lines.

- By increasing conductor spacing. By increasing the conductor spacing, the voltage at which corona happens is increased and therefore corona effects can be eliminated. Nevertheless, spacing cannot be increased too much otherwise the cost of supporting structure (for example, bigger cross arms and supports) may increase to a considerable extent.

Example 13. A 3-phase line has conductors 2 cm in diameter spaced equilaterally 1 m apart. If the air dielectric strength is 30 kV (max) per cm, calculate the line disruptive critical voltage. Take air density factor $\delta=0.952$ and irregularity factor $m_0=0.9$.

- Solution.

Conductor radius, $r=2/2=1$ cm

Conductor spacing, $d=1$ m=100 cm

Air dielectric strength, $q_0 = 30 \frac{kV}{cm} (max.) = 21.2 \text{ kV (rms) per cm}$

Disruptive critical voltage, $V_c = m_0 q_0 \delta r \log_e \left(\frac{d}{r} \right) \text{ kV} - \text{phase (rms value)}$

$$= 0.9 \times 21.2 \times 0.952 \times 1 \times \log_e \frac{100}{1} = 83.64 \text{ kv/phase}$$

Line voltage (RMS) $= \sqrt{3} \times 83.64 = 144.8 \text{ kV}$

Example 14. A 132 kV line with 1.956 cm diameter conductors is built so that corona happens if the line voltage surpasses 210 kV (RMS). If the value of potential gradient at which ionisation happens can be taken as 30 kV per cm, calculate the conductor spacing.

- Solution.

Assume the line is 3-phase.

Conductor radius, $r = 1.956/2 = 0.978$ cm

Air dielectric strength, $q_0 = \frac{30}{\sqrt{2}} = 21.2$ kV (rms) per cm

Disruptive voltage/phase, $V_c = \frac{210}{\sqrt{3}} = 121.25$ kV

Consider smooth conductors (for example, irregularity factor $m_0=1$) and standard pressure and temperature for which air density factor $\delta=1$. Let d cm be conductor spacing.

Disruptive voltage (RMS) per phase is

$$\begin{aligned}V_c &= m_0 q_0 \delta r \log_e \left(\frac{d}{r} \right) \text{ kV} \\&= 1 \times 21.2 \times 1 \times 0.978 \times \log_e \left(\frac{d}{r} \right) \\121.25 &= 20.733 \log_e \left(\frac{d}{r} \right) \\ \log_e \frac{d}{r} &= \frac{121.25}{20.733} = 5.848 \\ 2.3 \log_{10} \frac{d}{r} &= 5.848 \\ \log_{10} \frac{d}{r} &= \frac{5.848}{2.3} = 2.5426 \\ \frac{d}{r} &= \text{Antilog } 2.5426 \\ \frac{d}{r} &= 348.8\end{aligned}$$

Conductor spacing, $d = 348.8 \times r = 348.8 \times 0.978 = 341$ cm

Example 15. A 3-phase, 220 kV, 50 Hz transmission line has 1.5 cm radius conductor spaced 2 metres apart in equilateral triangular formation. If the temperature is 40°C

and atmospheric pressure is 76 cm, find the corona loss per km of the line. Consider $m_0=0.85$.

- Solution.

The corona loss is expressed as:

$$P = \frac{242.2}{\delta} (f + 25) \sqrt{\frac{r}{d}} (V - V_c)^2 \times \frac{10^{-5} kW}{km} / phase$$

$$\delta = \frac{3.92b}{273 + t} = \frac{3.92 \times 76}{273 + 40} = 0.952$$

Assuming $q_0 = 21.2 \frac{kV}{cm} (rms)$

Critical disruptive voltage per phase is

$$\begin{aligned} V_c &= m_0 q_0 \delta r \log_e \frac{d}{r} \text{ kV} \\ &= 0.85 \times 21.2 \times 0.952 \times 1.5 \times \log_e \frac{200}{1.5} = 125.9 \text{ kV} \end{aligned}$$

Supply voltage per phase, $V = \frac{220}{\sqrt{3}} = 127 \text{ kV}$

Replacing the above values, corona loss can be expressed as:

$$\begin{aligned} P &= \frac{242.2}{0.952} (50 + 25) \times \sqrt{\frac{1.5}{200}} \times (127 - 125.9)^2 \times \frac{10^{-5} kW}{phase} / km \\ &= \frac{242.2}{0.952} \times 75 \times 0.0866 \times 1.21 \times 10^{-5} \frac{kW}{km} / phase \\ &= 0.01999 \frac{kW}{km} / phase \end{aligned}$$

Complete corona loss per km for three phases

$$= 3 \times 0.01999 \text{ kW} = 0.05998 \text{ kW}$$

Example 16. A certain 3-phase equilateral transmission line has an overall corona loss of 53 kW at 106 kV and a loss of 98 kW at 110.9 kV. Calculate the disruptive

critical voltage. What is the corona loss at 113 kV?

- Solution.

The power loss due to corona for 3 phases is expressed as:

$$P = 3 \times \frac{242.2(f + 25)}{\delta} \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

As f, δ, r, d are the same for the two cases,

$$P \propto (V - V_c)^2$$

For first case, $P=53 \text{ kW}$ and $V = \frac{106}{\sqrt{3}} = 61.2 \text{ kV}$

For second case, $P=98 \text{ kW}$ and $V = \frac{110.9}{\sqrt{3}} = 64 \text{ kV}$

$$53 \propto (61.2 - V_c)^2$$

$$98 \propto (64 - V_c)^2$$

Dividing the last two formulas, it can be found:

$$\frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2}$$

$$V_c = 54 \text{ kV}$$

Let W kilowatt be the power loss at 113 kV

$$W \propto \left(\frac{113}{\sqrt{3}} - V_c \right)^2$$

$$\propto (65.2 - 54)^2$$

Dividing the last two formulas, it can be found:

$$\frac{W}{53} = \frac{(65.2 - 54)^2}{(61.2 - 54)^2}$$

$$W = \left(\frac{11.2}{7.2} \right)^2 \times 53 = 128 \text{ kW}$$

Overhead Line Sag

While building an overhead line, it is crucial that conductors are under safe tension. If the conductors are too stretched between supports in an attempt to save conductor material, the stress in the conductor may reach critical value and in some cases the conductor may break due to excessive tension. In order to secure conductor safe tension, they are not completely stretched but are allowed to have a dip or sag. The difference in level between support points and the conductor lowest point is called sag. Figure 23 (a) presents a conductor suspended between two equilevel supports A and B. The conductor is not completely stretched but is allowed to have a dip. The conductor lowest point is O and the sag is S. The following items can be noted:

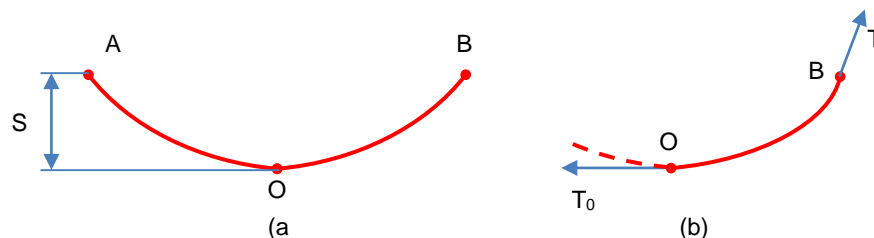


Figure 23. Conductor suspension between two supports

- When the conductor is suspended between two supports at the same level, it forms the shape of catenary. Nevertheless, if the sag is very small in comparison with the span, then sag-span curve is like a parabola.
- The tension at any point on the conductor acts tangentially. Therefore, tension T_0 at the lowest point O acts horizontally as presented in Figure 23 (b).
- The horizontal tension component is constant throughout the wire length.
- The tension at supports is roughly equal to the horizontal tension acting at any point on the wire. Therefore, if T is the tension at the support B, then $T=T_0$.

Sag and tension of the conductor

This is an important point in the overhead line mechanical design. The conductor sag needs to be maintained to a minimum in order to decrease the required conductor

material and to avoid extra pole height for sufficient clearance above earth level. It is also preferable that conductor tension is low to avoid the conductor mechanical failure and to allow the use of less strong supports. Nevertheless, low conductor tension and minimum sag cannot be achieved. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Hence in reality, a compromise is made between the two.

Sag Calculation

In an overhead line, the sag has to be adjusted so that tension in the conductors is within safe boundaries. The tension is governed by conductor weight, wind effects, ice loading and temperature changes. It is a common practice to maintain conductor tension less than 50% of its ultimate tensile strength. For example, minimum safety factor in respect of conductor tension needs to be 2. We shall now find sag and conductor tension when (a) supports are at equal levels and (b) supports are at different levels.

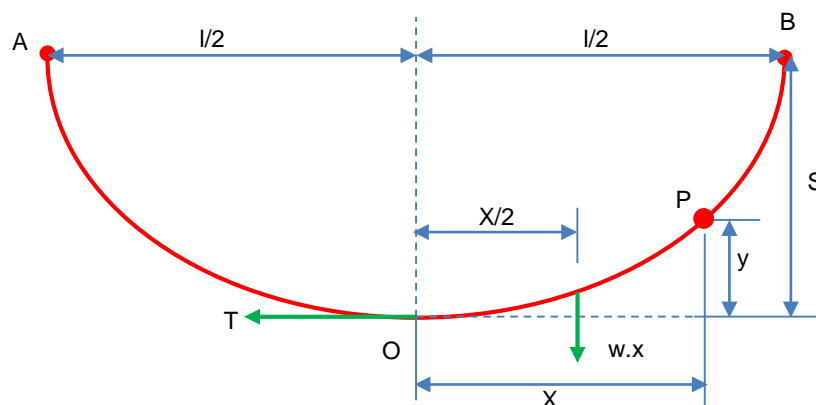


Figure 24. Conductor between two equilevel supports

- When supports are at same levels. Consider a conductor between two equilevel supports A and B with O as the lowest point as presented in Figure 24. It can be shown that lowest point will be at the mid-span. Consider:

l =Span length

w =Weight per conductor unit length

T =Tension in the conductor.

Consider a point P on the conductor. Considering the lowest point O as the origin, let the co-ordinates of point P be x and y. Assuming that the curvature is so small that curved length is equal to its horizontal projection (for example, OP=x), the two forces acting on the portion OP of the conductor are:

(a) The conductor weight wx acting at a distance $x/2$ from O.

(b) The tension T acting at O.

Equating the moments of above two forces about point O, we find:

$$Ty = wx \times \frac{x}{2}$$

$$y = \frac{wx^2}{2T}$$

The maximum dip (sag) is expressed by the value of y at either of the supports A and B. At support A, $x=l/2$ and $y=S$

$$\text{Sag, } S = \frac{w(l/2)^2}{2T} = \frac{wl^2}{8T}$$

- When supports are at different levels. In hilly locations, we typically come across conductors suspended between supports at different levels. Figure 25 presents a conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O.

Suppose:

l =Span length

h =Difference in levels between two supports

x_1 =Distance of support at lower level (for example, A) from O

x_2 =Distance of support at higher level (for example, B) from O

T =Conductor tension

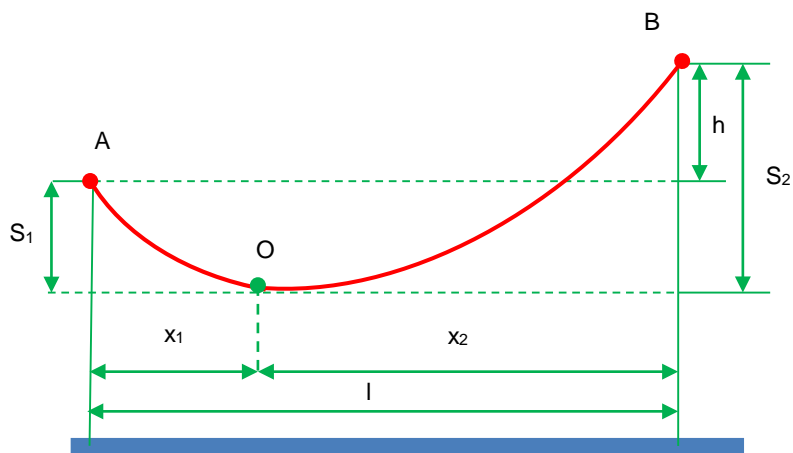


Figure 25. Conductor suspended between two different levels

If w is the conductor weight per unit length, then,

$$\text{Sag } S_1 = \frac{wx_1^2}{2T}$$

and

$$\text{Sag } S_2 = \frac{wx_2^2}{2T}$$

Also

$$x_1 + x_2 = l \quad (1)$$

Now

$$S_2 - S_1 = \frac{w}{2T} [x_2^2 - x_1^2] = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$S_2 - S_1 = \frac{wl}{2T} (x_2 - x_1) \quad x_1 + x_2 = l$$

But

$$S_2 - S_1 = h$$

$$h = \frac{wl}{2T}(x_2 - x_1)$$

Or

$$x_2 - x_1 = \frac{2Th}{wl} \quad (2)$$

Solving expressions (1) and (2), it can be found:

$$x_1 = \frac{l}{2} - \frac{Th}{wl}$$

$$x_2 = \frac{l}{2} + \frac{Th}{wl}$$

Having determined x_1 and x_2 , values of S_1 and S_2 can be easily found.

Wind and ice loading effect

The above equations for sag are correct only in still air and at normal temperature when the conductor is acted only by its weight only. Nevertheless, in real life a conductor may have ice coating and simultaneously exposed to wind pressure. The weight of ice acts vertically downwards for example, in the same direction as the conductor weight. The force due to the wind is assumed to act horizontally for example, at right angle to the conductor projected surface. Therefore, the complete force on the conductor is the vector sum of horizontal and vertical forces as presented in Figure 26 (c).

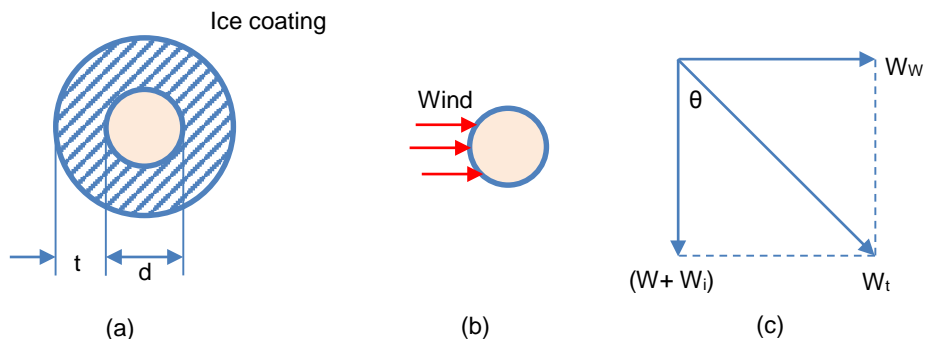


Figure 26. Wind effect on the conductor

Overall weight of conductor per unit length is:

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

Where

w - conductor weight per unit length (conductor material density x volume per unit length)

w_i - ice weight per unit length (density of ice x volume of ice per unit length)

w_w - wind force per unit length (wind pressure per unit area x projected area per unit length)

When the conductor has wind and ice loading, the following points have to be considered:

- The conductor sets itself in a plane at an angle to the vertical where

$$\tan \theta = \frac{w_w}{w + w_i}$$

- The sag in the conductor is expressed as:

$$S = \frac{w_t l^2}{2T}$$

Therefore, S represents the slant sag in a direction making an angle to the vertical. If no specific mention is made in the problem, then slant sag is found by using the above equation.

- The vertical sag = $S \cos \theta$

Example 17. A 132 kV transmission line has the following parameters:

Weight of conductor = 680 kg/km; Length of span = 260 m

Overall strength = 3100 kg; Safety factor = 2

Find the height above earth at which the conductor needs to be supported. Required earth clearance is 10 metres.

- Solution.

Weight of conductor/metre run, $w = 680/1000 = 0.68 \text{ kg}$

$$\text{Working tension, } T = \frac{\text{Ultimate strength}}{\text{Safety factor}} = \frac{3100}{2} = 1550 \text{ kg}$$

Span length, $l = 260 \text{ m}$

$$\text{Sag} = \frac{wl^2}{8T} = \frac{0.68 \times 260^2}{8 \times 1550} = 3.7 \text{ m}$$

Conductor has to be supported at a height of $10+3.7=13.7 \text{ m}$

Example 18. A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm^2 . The tension in the conductor is 2000 kg. If the conductor material specific gravity is 9.9 gm/cm^3 and wind pressure is 1.5 kg/m length, find the sag. Calculate also the vertical sag.

- Solution.

Span length, $l=150 \text{ m}$; Working tension, $T=2000 \text{ kg}$

Wind force/m length of conductor, $w_w=1.5 \text{ kg}$

Weight of conductor/m length, $w=\text{Specific Gravity} \times \text{Volume of 1 m conductor}$

$$= 9.9 \times 2 \times 100 = 1980 \text{ gm} = 1.98 \text{ kg}$$

Overall weight of 1 m length of conductor is:

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{1.98^2 + 1.5^2} = 2.48 \text{ kg}$$

$$\text{Sag, } S = \frac{w_t l^2}{8T} = \frac{2.48 \times 150^2}{8 \times 2000} = 3.48 \text{ m}$$

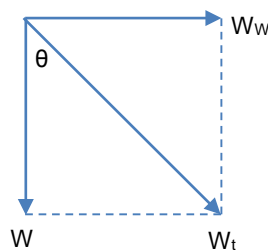


Figure 27. Overall conductor weight

This is the value of slant sag in a direction making an angle with the vertical. Going back to Figure 27, the value of θ is expressed as:

$$\tan \theta = \frac{w_w}{w} = \frac{1.5}{1.98} = 0.76$$

$$\theta = \tan^{-1}0.76 = 37.23^\circ$$

$$\text{Vertical sag} = S \cos \theta = 3.48 \times \cos 37.23^\circ = 2.77 \text{ m}$$

Example 19. A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of 1.29 cm^2 , weighs 1170 kg/km and has a breaking stress of 4218 kg/cm^2 . Find the sag for a safety factor of 5, allowing a wind pressure of 122 kg per square metre of projected surface. Calculate the vertical sag.

- Solution.

Span length, $l=200 \text{ m}$

Weight of conductor/m length, $w=1170/1000=1.17 \text{ kg}$

$$\text{Working tension, } T = \frac{4218 \times 1.29}{5} = 1088 \text{ kg}$$

$$\text{Conductor diameter, } d = \sqrt{\frac{4 \times \text{area}}{\pi}} = \sqrt{\frac{4 \times 1.29}{\pi}} = 1.28 \text{ cm}$$

$$\begin{aligned} \text{Wind force/m length, } w_w &= \text{Pressure} \times \text{projected area in m}^2 \\ &= 122 \times (1.28 \times 10^{-2} \times 1) = 1.56 \text{ kg} \end{aligned}$$

Overall weight of conductor per metre length is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95 \text{ kg}$$

Total weight of conductor per metre length is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95 \text{ kg}$$

$$\text{Slant sag, } S = \frac{w_t l^2}{8T} = \frac{1.95 \times 200^2}{8 \times 1088} = 8.96 \text{ m}$$

The slant sag makes an angle θ with the vertical where value of θ expressed as:

$$\theta = \tan^{-1} \left(\frac{w_w}{w} \right) = \tan^{-1} \left(\frac{1.56}{1.17} \right) = 53.13^\circ$$

$$\text{Vertical sag} = S \cos \theta = 8.96 \times \cos 53.13^\circ = 5.37 \text{ m}$$

Example 20. A transmission line has a span of 275 m between level supports. The conductor has an effective diameter of 1.96 cm and weighs 0.865 kg/m. Its overall strength is 8060 kg. If the conductor has ice coating of radial thickness 1.27 cm and is exposed to a wind pressure of 3.9 gm/cm² of projected surface, compute sag for a safety factor of 2. Weight of 1 c.c. of ice is 0.91 gm.

- Solution.

Span length, $l=275$ m ; Weight of conductor/m length, $w=0.865$ kg

Conductor diameter, $d=1.96$ cm; Ice coating thickness, $t=1.27$ cm Working tension, $T=8060/2=4030$ kg Volume of ice per metre (for example, 100 cm) length of conductor

$$\begin{aligned} &= \pi t(d + t) \times 100 \text{ cm}^3 \\ &= \pi \times 1.27 \times (1.96 + 1.27) \times 100 = 1288 \text{ cm}^3 \end{aligned}$$

Weight of ice per metre length of conductor is:

$$w_i = 0.91 \times 1288 = 1172 \text{ gm} = 1.172 \text{ kg}$$

Wind force/m length of conductor is:

$$\begin{aligned} w_w &= [\text{Pressure}] \times [(d + 2t) \times 100] \\ &= [3.9] \times (1.96 + 2 \times 1.27) \times 100 \text{ gm} = 1755 \text{ gm} = 1.755 \text{ kg} \end{aligned}$$

Overall weight of conductor per metre length of conductor is

$$\begin{aligned} w_t &= \sqrt{(w + w_i)^2 + (w_w)^2} \\ &= \sqrt{(0.865 + 1.172)^2 + (1.755)^2} = 2.688 \text{ kg} \end{aligned}$$

$$\text{Sag} = \frac{w_t l^2}{8T} = \frac{2.688 \times 275^2}{8 \times 4030} = 6.3 \text{ m}$$

Example 21. A transmission line has a span of 214 metres between level supports. The conductors have a cross-sectional area of 3.225 cm². Find the factor of safety under the following circumstances:

Vertical sag=2.35 m; Wind pressure=1.5 kg/m run

Breaking stress=2540 kg/cm²; Weight of conductor=1.125 kg/m run

- Solution.

Here, l=214 m; w=1.125 kg; w_w=1.5 kg

Overall weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1.125)^2 + 1.5^2} = 1.875 \text{ kg}$$

If f is the safety factor, then,

$$\text{Working tension, } T = \frac{\text{Breaking stress} \times \text{conductor area}}{\text{safety factor}} = \frac{2540 \times 3.225}{f} = \frac{8191}{f} \text{ kg}$$

$$\text{Slant Sag, } S = \frac{\text{Vertical sag}}{\cos \theta} = \frac{2.35 \times 1.875}{1.125} = 3.92 \text{ m}$$

$$\text{Now } S = \frac{w_t l^2}{8T}$$

$$\text{Or } T = \frac{w_t l^2}{8S}$$

$$\frac{8191}{f} = \frac{1.875 \times 214^2}{8 \times 3.92}$$

$$\text{Or safety factor } f = \frac{8191 \times 8 \times 3.92}{1.875 \times 214^2} = 3$$

Example 22. Transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm². The overall strength is 5000 kg/cm² and safety factor is 5. The material specific gravity is 8.9 gm/cc. The wind pressure is 1.5 kg/m. Find the conductor height above the earth level at which it has to be supported if a minimum clearance of 7 m is to be left between the earth and the conductor.

- Solution.

Span length, $l=150$ m; Wind force/m run, $w_w=1.5$ kg

Weight of conductor/m run $w = \text{conductor area} \times 100 \text{ cm} \times \text{sp. gravity}$

$$= 2 \times 100 \times 8.9 = 1780 \text{ gm} = 1.78 \text{ kg}$$

Working tension, $T = 5000 \times \frac{2}{5} = 2000 \text{ kg}$

Overall weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{1.78^2 + 1.5^2} = 2.33 \text{ kg}$$

$$\text{Slant sag, } S = \frac{w_t l^2}{8T} = \frac{2.33 \times 150^2}{8 \times 2000} = 3.28 \text{ m}$$

$$\text{Vertical sag} = S \cos \theta = 3.28 \times \frac{w}{w_t} = 3.28 \times \frac{1.78}{2.33} = 2.5 \text{ m}$$

Conductor has to be supported at a height of $7+2.5=9.5$ m

Example 23. The towers of height 30 m and 90 m respectively support a overhead line conductor at water crossing. The horizontal distance between the towers is 500 m. If the tension in the conductor is 1600 kg, calculate the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of conductor is 1.5 kg/m. Bases of the towers can be considered to be at water level.

- Solution.

Figure 28 presents the conductor suspended between two supports A and B at different levels with O as the lowest point on the conductor. Here, $l=500$ m; $w=1.5$ kg;

$T=1600$ kg. Difference in support levels, $h=90-30=60$ m. Let the lowest point O of the conductor be at a distance x_1 from the support at lower level (for example, support A) and at a distance x_2 from the support at higher level (for example, support B).

Apparently, $x_1+x_2=500$ m

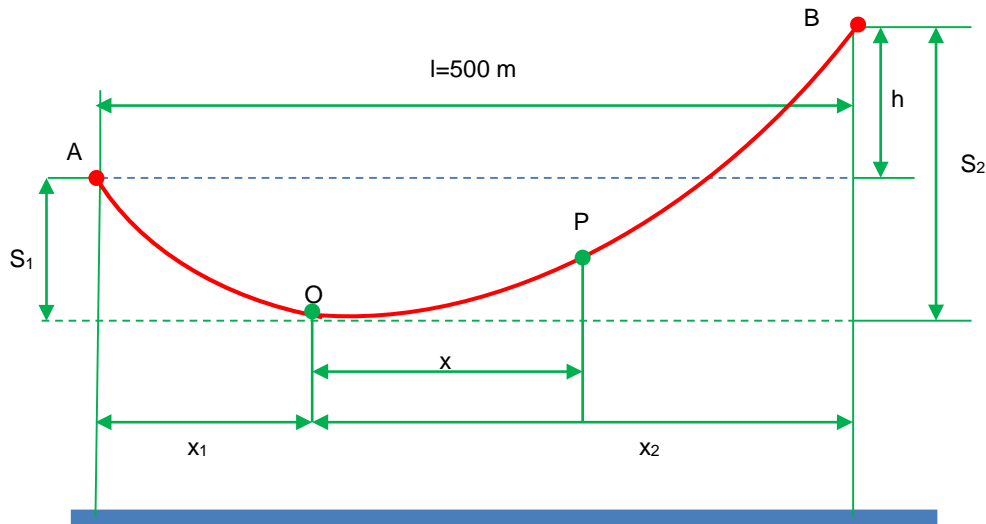


Figure 28. Conductor suspended between two different levels

Now
$$S_1 = \frac{wx_1^2}{2T} \quad \text{and} \quad S_2 = \frac{wx_2^2}{2T}$$

$$h = S_2 - S_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T}$$

Or

$$60 = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$x_2 - x_1 = \frac{60 \times 2 \times 1600}{1.5 \times 500} = 256 \text{ m}$$

Solving above formulas it can be found: $x_1 = 122$ m, $x_2 = 378$ m

Now,
$$S_1 = \frac{wx_1^2}{2T} = \frac{1.5 \times 122^2}{2 \times 1600} = 7 \text{ m}$$

Lowest point O clearance from water level = $30 - 7 = 23$ m

Let the mid-point P be at a distance x from the lowest point O.

Apparently,
$$x = 250 - x_1 = 250 - 122 = 128 \text{ m}$$

Sag at mid-point P,
$$S_{mid} = \frac{wx^2}{2T} = \frac{1.5 \times 128^2}{2 \times 1600} = 7.68 \text{ m}$$

Clearance of mid-point P from water level = 23 + 7.68 = 30.68 m

Example 24. An overhead transmission line conductor with a parabolic arrangement weighs 1.925 kg per metre of length. The area of cross-section of the conductor is 2.2 cm² and the overall strength is 8000 kg/cm². The supports are 600 m apart having 15 m difference of levels. Find the sag from the taller of the two supports which have to be allowed so that the safety factor shall be 5. Consider that ice load is 1 kg per metre run and there is no wind pressure.

- Solution. Figure 29 presents the conductor suspended between two supports at A and B at different levels with O as the lowest point on the conductor.

Here, l=600 m; w_i=1 kg; h=15m

w=1.925 kg; T=8000×2.2/5=3520 kg

Complete weight of 1 m length of conductor is:

w_t=w+w_i=1.925+1=2.925 kg

Let the conductor lowest point O be at a distance x₁ from the support at lower level (for example, A) and at a distance x₂ from the support at higher level (for example, B).

$$\text{Apparently, } x_1 + x_2 = 600 \text{ m} \quad (1)$$

Now,
$$h = S_2 - S_1 = \frac{w_t x_2^2}{2T} - \frac{w_t x_1^2}{2T}$$

Or

$$15 = \frac{w_t}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$x_2 - x_1 = \frac{2 \times 15 \times 3520}{2.925 \times 600} = 60 \text{ m} \quad (2)$$

Solving formulas (1) and (2), it can be found, x₁=270 m and x₂=330 m

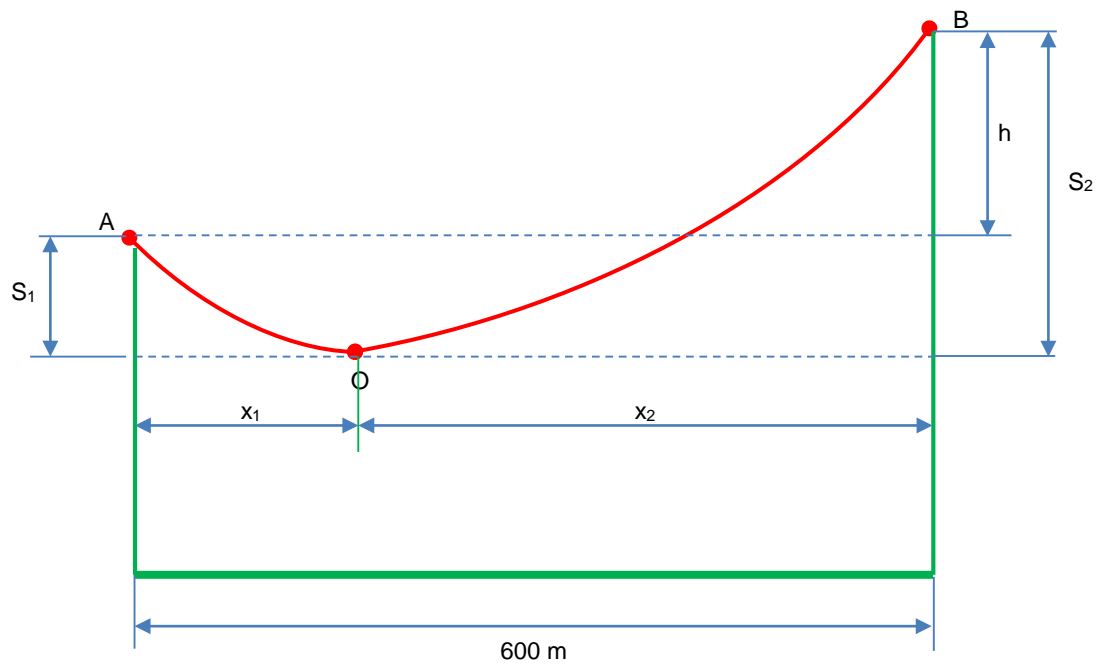


Figure 29. Conductor suspended between two different levels

Sag from the taller of the two towers is

$$S_2 = \frac{w_t x_2^2}{2T} = \frac{2.925 \times 330^2}{2 \times 3520} = 45.24 \text{ m}$$

Example 25. An overhead line at a river crossing is supported from two towers at heights of 40 m and 90 m above water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 2000 kg, calculate the clearance between the conductor and water at a point mid-way between the towers. Conductor weight is 1 kg/m.

- Solution. Figure 30 shows the complete configuration.

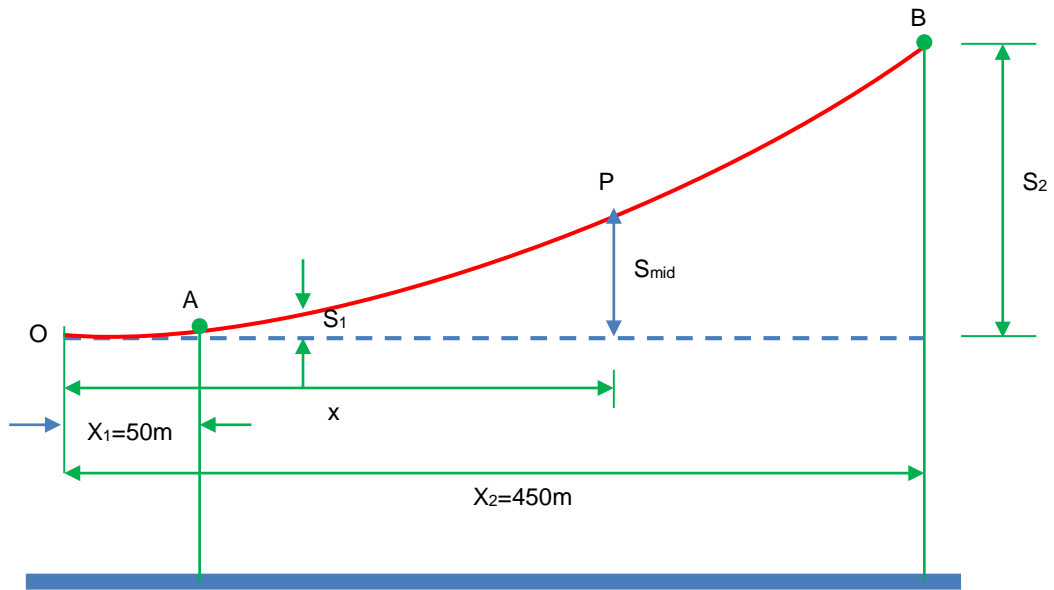


Figure 30. Overhead line at a river crossing

Here, $h=90-40=50$ m; $l=400$ m

$T=2000$ kg; $w=1$ kg/m

$$\text{Apparently, } x_1+x_2=400 \text{ m} \quad (1)$$

Now:

$$h = S_2 - S_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T}$$

Or

$$50 = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$x_2 - x_1 = \frac{50 \times 2 \times 2000}{400} = 500 \text{ m} \quad (2)$$

Solving expressions (1) and (2), it can be found, $x_2=450$ m and $x_1=-50$ m

Now x_2 is the distance of higher support B from the conductor lowest point O, whereas x_1 is that of lower support A. As the span is 400 m, hence, point A lies on the same side of O as B (as presented in Figure 30). Horizontal distance of mid-point P from lowest point O is

$x = \text{Distance of A from O} + 400/2 = 50 + 200 = 250$ m

Sag at point P,
$$S_{mid} = \frac{wx^2}{2T} = \frac{1 \times 250^2}{2 \times 2000} = 15.6 \text{ m}$$

Now Sag
$$S_2 = \frac{wx_2^2}{2T} = \frac{1 \times 450^2}{2 \times 2000} = 50.6 \text{ m}$$

Height of point B above mid-point P:

$$= S_2 - S_{mid} = 50.6 - 15.6 = 35 \text{ m}$$

Clearance of mid-point P above water level = $90 - 35 = 55 \text{ m}$

Example 26. An overhead line over a hillside with the gradient of 1:20, is supported by two 22 m high towers with a distance of 300 m between them. The lowest conductor is fixed 2 m below the top of each tower. Calculate the clearance of the conductor from the earth. Given that conductor weighs 1 kg/m and the allowable tension is 1500 kg.

- Solution. The conductors are supported between towers AD and BE over a hillside having gradient of 1:20 as presented in Figure 31. The lowest point on the conductor is O and $\sin\theta = 1/20$.

Effective height of each tower (AD or BE) = $22 - 2 = 20 \text{ m}$

Vertical distance between towers is:

$$h = EC = DE \sin \theta = 300 \times \frac{1}{20} = 15 \text{ m}$$

Horizontal distance between two towers is:

$$DC = \sqrt{DE^2 - EC^2} = \sqrt{300^2 - 15^2} \sim 300 \text{ m} \quad (1)$$

Or

$$x_1 + x_2 = 300 \text{ m}$$

Now

$$h = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T} = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

Or

$$x_2 - x_1 = \frac{2Th}{w(x_2+x_1)} = \frac{2 \times 1500 \times 15}{1 \times 300} = 150 \text{ m} \quad (2)$$

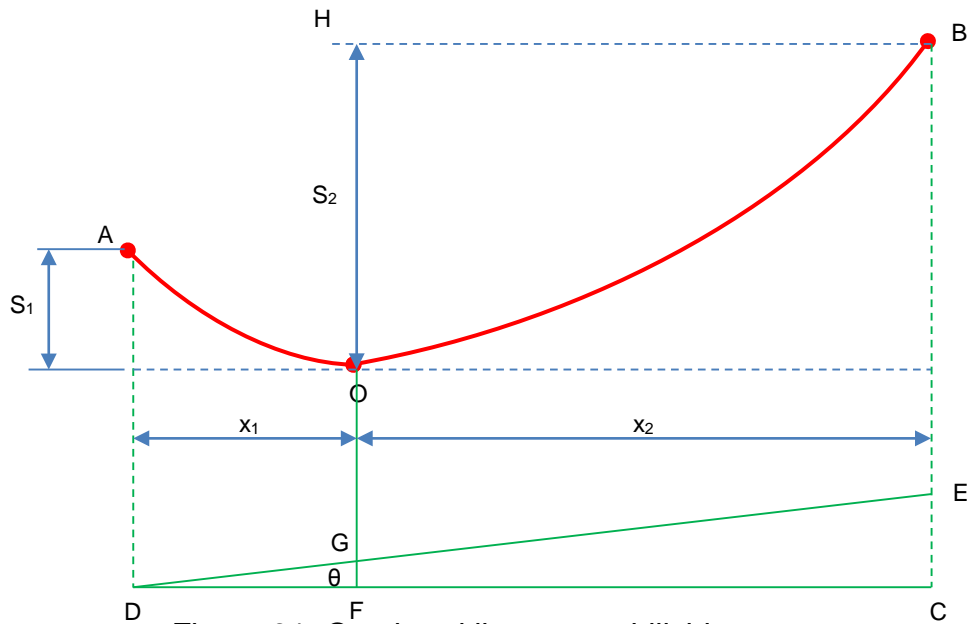


Figure 31. Overhead line over a hillside

Solving expression (1) and (2), it can be found $x_1=75 \text{ m}$ and $x_2=225 \text{ m}$

$$\text{Sag } S_2 = \frac{wx_2^2}{2T} = \frac{1 \times 225^2}{2 \times 1500} = 16.87 \text{ m}$$

Now

$$BC = BE + EC = 20 + 15 = 35 \text{ m}$$

Clearance of the lowest point O from the earth is:

$$\begin{aligned} OG &= HF - S_2 - GF \\ &= BC - S_2 - GF \end{aligned}$$

Now

$$\begin{aligned} GF &= x_1 \tan \theta = 75 \times 0.05 = 3.75 \text{ m} \\ &= 35 - 16.87 - 3.75 = 14.38 \text{ m} \end{aligned}$$

Example 27. A transmission tower on a level earth gives a minimum clearance of 8 metres for its lowest conductor with sag of 10 m for a span of 300 m. If the same tower is to be used over a slope of 1 in 15, calculate the minimum earth clearance obtained for the same span, same conductor and same weather conditions.

- Solution. On level earth

Sag,
$$S = \frac{wl^2}{8T}$$

$$\frac{w}{T} = \frac{8S}{l^2} = \frac{8 \times 10}{300^2} = \frac{8}{9 \times 10^3}$$

Height of tower=Sag+Clearance=10+8=18 m

On sloping earth. The conductors are supported between towers AD and BE over a sloping earth having a gradient 1 in 15 as presented in Figure 32. The height of each tower (AD or BE) is 18 m. Vertical distance between the two towers is

$$h=EC=DE \sin\theta=300 \times 1/15=20 \text{ m}$$

$$\text{Now } x_1+x_2=300 \text{ m} \quad (1)$$

Also

$$h = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T} = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$x_2 - x_1 = \frac{2Th}{w(x_2+x_1)} = \frac{2 \times 9 \times 10^3 \times 20}{8 \times 300} = 150 \text{ m} \quad (2)$$

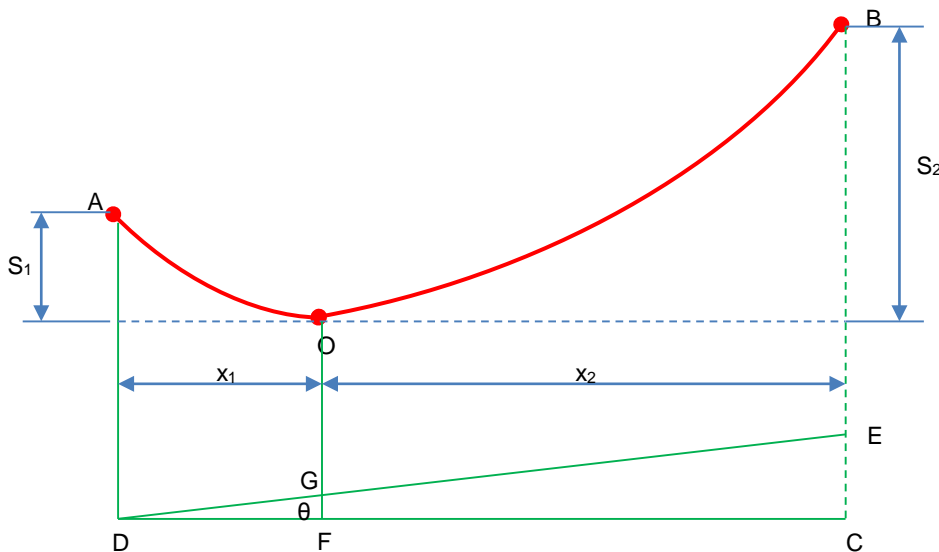


Figure 32. The conductors supported between towers over a sloping ground

Solving formulas (1) and (2), it can be found $x_1=75 \text{ m}$ and $x_2=225 \text{ m}$

Now

$$S_1 = \frac{wx_1^2}{2T} = \frac{8 \times 75^2}{2 \times 9 \times 10^3} = 2.5 \text{ m}$$

$$S_2 = \frac{wx_2^2}{2T} = \frac{8 \times 225^2}{2 \times 9 \times 10^3} = 22.5 \text{ m}$$

Point O clearance from the earth is:

$$OG = BC - S_2 - GF = 38 - 22.5 - 5 = 10.5 \text{ m}$$

$$GF = x_1 \tan\theta = 75 \times 1/15 = 5 \text{ m]$$

Since O is the origin, the equation of slope of earth is expressed as:

$$y = mx + A$$

Here:

$$m = \frac{1}{15} \text{ and } A = OG = -10.5 \text{ m}$$

$$y = \frac{x}{15} - 10.5$$

Clearance C from the earth at any point x is:

$$\begin{aligned} C = \text{Equation of conductor curve} - y &= \left(\frac{wx^2}{2T} \right) - \left(\frac{x}{15} - 10.5 \right) \\ &= \frac{8x^2}{2 \times 9 \times 10^3} - \left(\frac{x}{15} - 10.5 \right) = \frac{x^2}{2250} - \frac{x}{15} + 10.5 \end{aligned}$$

Clearance will be minimum when $\frac{dC}{dx} = 0$ for example,

$$\frac{d}{dx} \left[\frac{x^2}{2250} - \frac{x}{15} + 10.5 \right] = 0$$

Or

$$\frac{2x}{2250} - \frac{1}{15} = 0$$

Or

$$x = \frac{1}{15} \times \frac{2250}{2} = 75 \text{ m}$$

For example, minimum clearance will be at a point 75 m from O.

$$\text{Minimum clearance} = \frac{x^2}{2250} - \frac{x}{15} + 10.5 = \frac{75^2}{2250} - \frac{75}{15} + 10.5 = 2.5 - 5 + 10.5 = 8 \text{ m}$$

Certain Mechanical Principles

Mechanical safety factors to be used in overhead line design should depend to some degree on the continuity of operation importance. In principle, the line strength needs to be such as to provide against the worst expected weather conditions. Some important issues in the overhead transmission lines mechanical design are:

- Tower height: It is dependent upon the span length. With long spans, relatively few towers are needed but they have to be tall and correspondingly costly. Typically, it is not possible to find the tower height and span length on the basis of direct construction costs because the lightning hazards highly increase as the height of the conductors above earth is increased. This is one reason that horizontal spacing is preferred in spite of the wider right of way.
- Conductor clearance to earth: The conductor clearance to earth at the time of biggest sag should not be less than some specified distance (typically between 6 and 12 m), depending on the voltage, on the nature of the country and on the local regulations. The biggest sag may happen on the hottest day of summer on account of the expansion of the wire or it may happen in winter owing to the formation of a heavy coating of ice on the wires. Specific provisions must be made for melting ice from the power lines.
- Sag and tension: When installing overhead transmission lines, it is mandatory to allow a reasonable safety factor in respect of the tension to which the conductor is subjected. The tension is governed by the wind effects, ice loading and temperature changes. The relationship between tension and sag depends on the loading conditions and temperature changes. For instance, the tension increases when the temperature reduces and there is a corresponding sag reduction. Icing-up of the line and wind loading will cause conductor stretching by an amount dependent on the line tension.

In sag planning, tension and clearance to earth of a given span, a maximum stress is chosen. It is then aimed to have this stress developed at the worst probable weather conditions (for example, minimum expected temperature, maximum ice loading and maximum wind). Wind loading increases the sag in the direction of resultant loading

but reduces the vertical component. Hence, in clearance calculations, the wind effect should not be included unless horizontal clearance is important.

- Stringing charts: For use in the field work of stringing the conductors, temperature-sag and temperature tension charts are printed for the given conductor and loading conditions. Such curves are known as stringing charts and are shown in Figure 33. These charts are very useful while stringing overhead transmission lines.
- Conductor spacing: Conductor spacing has to be such to provide safety against flash-over when the wires are swinging in the wind. The adequate spacing is a function of span length, voltage and weather conditions. The use of horizontal spacing removes the danger caused by unequal ice loading. Small wires or light material wires are exposed to more swinging by the wind than heavy conductors. Hence, light wires need to be provided bigger spacings.
- Conductor vibration: Wind exerts pressure on the exposed conductor area. If the wind velocity is small, the conductor swinging is harmless given the clearance is sufficiently big so that conductors do not approach within the sparking distance of each other. A totally different vibration type, known as dancing, is caused by the action of fairly strong wind on a wire covered with ice, when the ice coating happens to take a form which makes a good air-foil section. Then the complete span may sail up like a kite until it reaches the limit of its slack, stops with a jerk and falls or sails back. The harmful impact of these vibrations happens at the clamps or supports where the conductor suffers fatigue and finally breaks. In order to save the conductors, dampers are used.

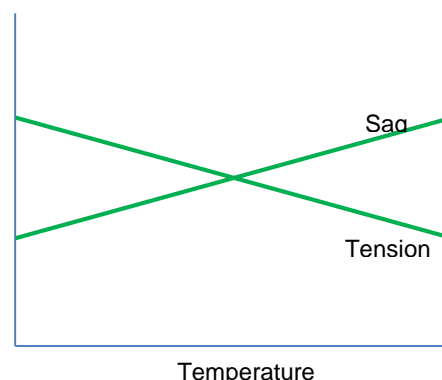


Figure 33. Stringing charts